

ON h -FUNCTION OF A BOUNDED SIMPLY CONNECTED REGION: DISC WITH DELETED DOUBLE SLITS

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The h -function or harmonic-measure distribution function for a region Ω with a specific fixed point z_0 in Ω , gives some information about the shape of the region. For a region Ω with a basepoint z_0 , we identify the set E_r , which is the intersection of the boundary of Ω and the closed disc of radius r centred at z_0 . The h -function $h(r)$ is given by the harmonic measure of the set E_r in Ω at z_0 . This function $h(r)$ only takes the values in the unit interval $[0,1]$, but $h(r)$ will take the value one only for the regions with bounded boundaries. This study is focused on the behaviour of the h -function(s) of a bounded simply connected regions Ω formed by deleting double slits from the unit disc centred at the origin when both slits lie on the real axis and vary in length. Three cases are considered: keep the size of both slits the same; keep the length of one of two slits as the radius of the disc; keep the length of one of two slits as it is bigger than the radius of the disc. For these regions Ω , the h -function will take the value 1 after some values of r . That is, the h -function $h(r)$ meets the line $y = 1$ at an angle ψ , subtended at the line $y = 1$ with the graph of $h(r)$ in the counterclockwise direction. We check how this angle ψ changes for the above three cases when the basepoint z_0 lies anywhere in between both slits inside the disc. For the first and third cases, when the basepoint z_0 moves between two slits from left to right along the real axis, the angle ψ increases from zero for a while and attains its maximum and then decreases to zero. In the first case, the maximum of the angle ψ has been attained at $\pi/2$. For the second case, when the basepoint z_0 moves between the two slits from left to right along the real line, the angle ψ decreases from $\pi/2$ to zero. These findings indicate that the h -function of these bounded regions Ω has interesting behaviour at the point r^* which is the furthest distance between the base point and the boundary of the region. Future research will focus on checking the behaviour at the same point r^* , when the basepoint varies along the imaginary axis within these regions Ω .

Keywords: Bounded region, h -function, Harmonic-measure, Simply connected region