

UNVEILING RAINBOW CONNECTIONS IN EXTENDED SANDAT GRAPHS

G.W.M.M.K. Dheerasinghe^{1*}, P.G.R.S. Ranasinghe² and A.A.I. Perera²

¹Department of Mathematics, University of Ruhuna, Sri Lanka

²Department of Mathematics, Faculty of Science, University of Peradeniya, Sri Lanka

*gwmkduor@gmail.com

Graph theory is a fundamental area of discrete mathematics, with graph colouring being one of its most captivating topics. In particular, edge colouring involves assigning colours to the edges of a graph, ensuring that no two adjacent edges have the same colour while using the fewest possible colours. A rainbow path in an edge-coloured graph is one where no two edges in the path have the same colour. If every pair of vertices in a graph is connected by at least one rainbow path, the graph is rainbow-connected. The minimum number of colours needed for a graph to be rainbow-connected is called the rainbow connection number ($rc(G)$). Map colouring, optimizing timetables, solving Sudoku puzzles, and minimum cost are some examples of the applications of graph colouring. Hence, it is important to study and introduce new graph classes. The present study introduced the comb product of the operation of cycle graph C_n and the higher-order extended version of the Sandat graph $SSt_m(n)$. The comb product of C_4 and $SSt_m(n)$ has been explored in detail. This comb product can be illustrated by connecting each vertex of C_4 to $SSt_m(n)$ such that the vertex set $V(C_4 \triangleright SSt_m(n)) = \{r_k, s_{ij}^h, t_i : 1 \leq k \leq 4, 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq h \leq m + 1\}$ and the edge set $E(C_4 \triangleright SSt_m(n)) = \{r_1r_2, r_2r_3, r_3r_4, r_4r_1\} \cup \{rt_i, rs_{ij}^h, t_i s_{ij}^1, s_{ij}^p s_{ij}^{p+1} : 1 \leq i \leq n, 1 \leq j \leq 2, 1 \leq h \leq m + 1, 1 \leq p \leq m\}$. An algorithm has been proposed for rainbow colouring of this graph aiming to establish the rainbow connection number. Future works will be conducted to confirm this rainbow connection number and introduce the comb product of the non-symmetric higher-order extended Sandat graphs. The rainbow connection number has practical applications in network design, such as in secure data transmission, where diverse routing paths help prevent interception and ensure reliability.

Keywords: Comb product, Cycle graphs, Edge colouring, Rainbow colouring, Sandat graphs