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Z-SETS IN LARGE SCALE GEOMETRY

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This study introduces an analogous version of the Z-set in large-scale geometry, inspired by its foundational role in infinite-dimensional topology. A closed subset $A \subseteq X$ is called Z-set of X, if there exist arbitrary small maps from X into $X \setminus A$; that is, for every open cover U of X, there exists a map from X into $X \setminus A$ which is U-close to the identity. Although the Z-set does not seem very appealing, it is the most central concept in infinite-dimensional topology. Extending this idea to large-scale geometry, we define Coarse Z-sets by analyzing their behavior under arbitrarily small maps of X into $X \setminus A$ and examining their structural properties in a global context. A subset $A \subseteq X$ is called Coarse Z-set if there exists a function from X into $X \setminus A$ that is “close” to identity map in the sense of large-scale geometry. Characterized by maps that are “close” to the identity, the Coarse Z-set can be thought of as a small set in a larger space; removing it does not change the overall structure of the original space. This study demonstrates that if a subset is a Coarse Z-set, the associated function is a quasi-isometry, guaranteeing coarse equivalence between the space and its complement. This equivalence preserves asymptotic dimensions, as expressed by $\mathrm{asdim}(X) = \mathrm{asdim}(X \setminus A)$. Furthermore, Coarse Z-sets are invariant under coarse equivalence, showcasing their robustness in large-scale geometry.

Keywords: Asymptotic dimension, Coarse equivalence, Coarse Z-set, Infinite-dimensional topology, Large-scale geometry, Quasi-isometry