

Choice of the Inverse Linear Mapping in the Method of Directly Defining the Inverse Mapping for Inhomogeneous Heat and Advection Problems

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Semi-analytical methods for solving nonlinear partial differential equations (PDEs) combine aspects of both analytical and numerical techniques. Unlike linear PDEs, which can be more straightforwardly solved using the separation of variables and superposition principles, nonlinear PDEs involve terms with nonlinear dependencies, making them significantly more complex to handle. The key to the analytical method lies in employing techniques like perturbation methods, similarity transformations, and integral transformations such as Fourier and Laplace transform to simplify the equations. These techniques often lead to obtaining exact solutions or approximations that provide valuable insights into the behavior of the system. However, due to the intricate nature of nonlinear PDEs, the analytical method may not always be applicable, and in these cases, numerical methods or approximate techniques become necessary. The Method of Directly Defining the inverse Mapping (MDDiM) is a semi-analytical method for obtaining approximate solutions for complicated nonlinear PDEs without using any kind of transformation. In this study, we explore different inverse linear mappings in order to determine the optimal inverse linear mapping that allows accuracy just after a few terms are evaluated by applying MDDiM to the inhomogeneous heat and advection problems. Additionally, we optimize the convergence control parameter by constructing an optimum control problem for minimizing the accumulated L^2 -norm of the squared residual errors using directed sum. This approach is far easier to compute than the square integrals of an infinite domain. By computing three-term optimal approximate solutions and comparing them with the exact solutions, we were able to choose the better inverse linear mapping for these two inhomogeneous PDEs. We can conclude that choosing a better inverse linear mapping leads to a more accurate solution in MDDiM.

Keywords: Advection Problems, Convergence control parameter, Heat Problem, Inverse linear mapping, Method of Directly Defining the inverse Mapping