

## INVARIANCE OF COARSE Z-SETS UNDER COARSE EMBEDDINGS

**P.D.D.A. Chandralal\* and A.K. Amarasinghe**

*Department of Mathematics, University of Peradeniya, Peradeniya, Sri Lanka.*

*\*dinushaakalanka330@gmail.com*

The  $Z$ -sets, as subsets of the Hilbert cube  $Q$ , are a central concept in infinite-dimensional topology. In this study, we extend the notion of  $Z$ -sets from infinite-dimensional topology to large-scale geometry, introducing the concept of Coarse  $Z$ -sets. A closed subset  $A \subseteq X$  is called a  $Z$ -set of  $X$  if for every open cover  $U$  of  $X$  and every function  $f \in C(Q, X)$  there exists a function  $g \in C(Q, X \setminus A)$  such that  $f$  and  $g$  are  $U$ -close. We define Coarse  $Z$ -sets by analysing the classical definition of  $Z$ -sets and examining their behavior under arbitrarily small maps from  $X$  into  $X \setminus A$ , but now in a global, coarse geometric context. Specifically, a subset  $A \subseteq X$  is called Coarse  $Z$ -set if there exists a function from  $X$  into  $X \setminus A$  that is “close” to the identity map in the sense of large-scale geometry. In the current work, we aim to redefine the Coarse  $Z$ -sets by using an analog of the Hilbert cube within large-scale geometry. We propose the Banach space  $l_\infty$  as the analogue version of the Hilbert cube within large-scale geometry. While investigating this idea, we show that Coarse  $Z$ -sets are invariant under coarse embeddings, if  $X$  coarsely embeds into  $Y$ , then the image of a Coarse  $Z$ -set of  $X$  under the embedding is a Coarse  $Z$ -set of  $Y$ . Consequently, the Coarse  $Z$ -set of  $Y$  can be identified once the Coarse  $Z$ -set of  $X$  is known. As a result, this demonstrates the connection between Coarse  $Z$ -set of a separable space and  $l_\infty$  since any separable space  $X$  can be coarsely embedded into the Banach space  $l_\infty$ . Thus, each Coarse  $Z$ -set of a separable space  $X$  can be mapped into  $l_\infty$  due to the universality of  $l_\infty$  for separable spaces.

**Keywords:** Coarse embedding, Hilbert cube, Infinite-dimensional topology, Large-scale geometry,  $Z$ -sets