

Actuarial Modeling for Epidemiological Diseases Spread in Sri Lanka: The Case of Dengue Fever

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Introduction

This study is done based on a developed actuarial model of Susceptible, Infected and Recovered (SIR) compartments, which describes the transfer dynamics in an insurance contract of a given population (Abramson, 2001). The SIR model created by Kermack and McKendrick (1927) set the mathematical and theoretical foundation for these epidemic models. Further research has been done to extended thresholds of these models using advanced analytical viewpoints (Mollison, 1995; Allen and Burgin, 2000; Kaddar et al., 2011; Bhattacharya et al., 2015).

The actuarial bases of epidemic disease spread are used with the intention of how to address the financial and economic possessions of such a venture. A book written by Slud (2001) provided imperative information of insurance and life annuity contracts. Feng (2005) developed an actuarial based model for epidemiology with the intention of building a bridge between epidemiology modeling and actuarial mathematics. His theory was utilized to design insurance contracts for the Great Plague in England and SARS epidemic in Hong Kong (Feng and Garrido, 2006). This was an imperative contribution made in literature of epidemic modeling which has open the gates of another testing ground for economic and financial analysts. In the context of Sri

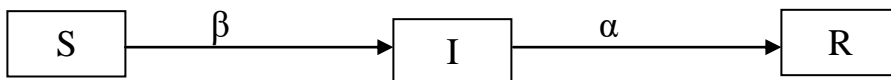
Lanka, few studies have been done for modeling the epidemic disease spread (Briët et al., 2008; Pathirana et al., 2009) and it is even more difficult to discover an analysis centered on actuarial based models. Hence, this study provides pioneering steps to the actuarial based model building for Sri Lankan epidemic profiles.

Objectives

While putting fore steps for the actuarial based modeling in relation to the epidemic disease spread in Sri Lanka, this study intends to revisit the theory by Feng (2005) and to obtain an expanded version of it based on SIR infection which describes the transfer dynamics in an insurance contract considering the highest total case recorded epidemics in Sri Lanka.

Methodology

A simple SIR model describes the conversion between sub-populations of susceptible, infectious and those who recovered. If recovery is permanent and recovered individuals are no longer susceptible to that pathogen then SIR model can be shown as follows,



β is the infecting rate for an individual per unit time and simultaneously α is the recovering rate from the diseases per unit time. SIRS model is more general than SIR model. The only difference when compared to SIR model is defining a new parameter called f which represents the rate of recovered individuals who are again susceptible per unit time due to the temporary recovery from the infectious disease.

Actuarial mathematics concepts are used to describe the financial transactions between two parties called insurer and insured.

Equivalence Principle:

$$\begin{aligned} E[\text{Present Value of Benefits}] \\ = E[\text{Present Value of Benefit Premiums}] \end{aligned}$$

For a continuous Whole Life Insurance Policy with a unit benefit the Level Premium Payment can be determined using equivalence principle as,

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} \quad (1)$$

Where \bar{A}_x is the actuarial present value of future benefit payments and \bar{a}_x is the actuarial present value of future premium payments. An actuarial based model has developed for the epidemiological diseases and the following equations are given for the annuity for hospitalization plan which has defined by using the whole life insurance policy. When δ is the force of interest, γ is the rate of recovering of susceptible (s) and infectious (i) compartments at time t ,

The total discounted future claim:

$$\bar{a}^i = \int_0^{\infty} e^{-\delta t} i(t) dt \quad (2)$$

The total discounted future premium:

$$\bar{a}^s = \int_0^{\infty} e^{-\delta t} s(t) dt \quad (3)$$

The force of infection:

$$\mu_t^s = \frac{-s'(t)}{s(t)} \quad (4)$$

The force of infection:

$$\mu_t^i = \frac{-i'(t)}{i(t)} \quad (5)$$

The level premium for the unit annuity for hospitalization plan:

$$\bar{P}(\bar{a}^i) = \frac{\bar{a}^i}{\bar{a}^s} = \frac{\delta \bar{a}^i}{1 - (\delta + \gamma) \bar{a}^i} \quad (6)$$

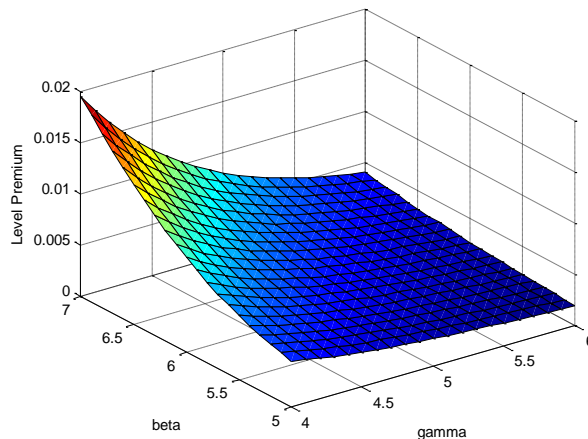
MATLAB statistical software and recorded epidemiological data from the official website of Epidemiological Unit, Sri Lanka are used as the materials of this study. Data were collected weekly for 40 weeks period beginning from 26th December 2015 to 30th September 2016.

Results and Discussion

Sensitivity of Level Premium Payment with respect to the parameters

Determining the level premium payment with positive benefit reserve is mainly focused when the actuarial model is developed. According to the observation of this study, there is an effect from the parameters, γ and β to determine the level premium payment.

Figure 1: Sensitivity of Premiums to Parameters



Source: Authors' Simulation using MATLAB

The rates taken at a monthly basis vary from 5-7 for infecting rate and 4-6 for recovering rate. Simulation shows a simultaneous decline in the recovery rate and increase in the infecting rate leaning the level premium rates towards zero. Premium rate reaches the highest possible level when a simultaneous increase in recovery rate and improvement

in infecting rate occur. Therefore, independent as well as simultaneous changes in the rates of getting infected and recovery specify the characteristics of level premium payment to be considered for a hospitalization plan. Above results were obtained while developing MATLAB simulation for the actuarial based model for SIR infectious disease developed by Feng (2005) for SARS epidemic.

Adjusting Level Premium Payment

According to retrospective approach the individual benefit reserve at time and t for the annuity for hospitalization plan with unit benefit can be formulated as follows,

$$\bar{V}_t(\bar{a}^i) = \int_0^t [\bar{P}(\bar{a}^i)s(t)e^{\delta t} - i(t)e^{\delta t}] dt, \quad t > 0 \quad (7)$$

However to satisfy the requirement of positivity of the benefit reserve curve, for all $t > 0$

$$\bar{P}(\bar{a}^i) \geq \frac{\int_0^t i(t)e^{\delta t} dt}{\int_0^t s(t)e^{\delta t} dt} \quad (8)$$

Feng (2005) has found out some results by setting up $\delta=0$ and those results do not make sense of the time value of money. Since the complexity of solving equations without neglecting the force of interest, an algorithm is defined and developed a MATLAB program through this study to calculate the minimum adjusted level premium for the hospitalization plan to satisfy the above condition. This program could be used to calculate the level premium of diseases which has a permanent immunity with the absence of Vector-Host transfer dynamics. Otherwise it will not be adequate to obtain 100 percent accuracy in results. Henceforth, it is important to identify the nature and characteristics of Sri Lankan epidemic diseases to recognize the applicability of the program developed.

Feasibility of SIR model to represent epidemics in Sri Lanka

This analysis is based on only the top 10 epidemics which have the highest number of total recorded cases for the selected period.

According to the data, highest recorded number of cases is Dengue and it is 72.65 % from the total top 10 epidemic cases. This implies that the probability of being infected by Dengue for a person is very high than the other diseases. However, there are considerable percentages for the diseases called Chickenpox (6.64%), Leptospirosis (5.44 %), Dysentery (4.83 %) and Typhus (3.29 %).

There are several patterns which can be seen when constructing time plots for the above 10 diseases. Some have clear seasonal patterns (Dengue fever). Also, some have very short-term fluctuations and it is difficult to determine the length of a season (Dysentery, Meningitis and etc.). Additionally, some diseases have declining patterns (Leptospirosis, Typhus and Leishmani). However, it is a huge area to study the reasons behind those patterns. Thus, this study is focused on developing an actuarial model for epidemiological diseases spread which can be used more generally to reduce the impact of several patterns. Dengue fever only contains a clear seasonal pattern based on the data for a 40-week period. APPENDIX provides further evidence on the seasonal behavior presence with dengue epidemic with a comparison of actual data with estimated measures for a given optimal lag length of 20 weeks for each season. Therefore, dengue fever has got expected seasonal features and it appears as a testing ground to practice feasibility of the insurance contract improved at the previous section of this study.

Actuarial Based Model for Dengue Fever Spread using SIR (Vector-Host) Model

There are some questions still to be addressed through further advancements of actuarial model considering long term effects such as Vector-Host transfer dynamics embedded with epidemic disease spread. According to the data it can be estimated the length of an epidemic season for some diseases such as dengue. But the SIR model defined by neglecting the type of disease which can be transferred by a vector. Dengue fever is the major epidemic disease in Sri Lanka which is generally spread by mosquitoes. Hence it is important to expand the

SIR model by including the Vector-Host transfer dynamics to find out an actuarial model for diseases such as Dengue fever. Using the same procedure carried out to obtain Result in 4.2 it can be easily shown that,

$$\bar{a}^{sh} = \frac{1 - (\delta + \gamma)\bar{a}^{ih}}{(\delta)} \quad (9)$$

and it yields to the level premium payment which formulated for the SIR infection model without the Vector-Host transfer dynamics being same here. Hence, it is reasonable to use the MATLAB program developed earlier through this study to calculate the minimum adjusted level premium for the hospitalization plan for Dengue fever.

Actuarial Model using SIRS Model

Moreover, other diseases have consisted of very short-term fluctuations and it is difficult to determine the length of the epidemic period. Also some people can be infected by the same disease more than once for the considered time period. On other hand, usually an insurance contract is drawn up for annum or a period of six months and it is rarely possible to adjust it with the epidemic season. Hence, the transformation within compartments for a long term can be described more generally using SIRS model than SIR model. But SIRS model is expressed using Delay Differential Equations and this study was not focused on simulating that model.

Conclusion and Policy Implications

This study is done based on a developed actuarial model of SIR infection which describes the transfer dynamics in an insurance contract in a given population. At the initial stage, we satisfied key assumptions and observed that the rate of infecting is positively related and the rate of recovering is negatively related to the level premium payment. Further, we developed a MATLAB program to calculate the minimum adjusted level premium for a hospitalization plan. Secondly this study obtained expanded models for the basic model to eliminate

some problems which occurred such as the Vector-Host relationship due to unsatisfied assumptions for real data. It is reasonable to expand the SIR model by including Vector-Host transfer dynamics to find out an actuarial model for Dengue fever, as it can be estimated for the length of an epidemic season for Dengue for the sample period. Results show that there is no impact from Vector-Host to determine the level premium payment. Finally, we suggest the SIRS infection model with delayed differential equations as an appropriate solution which arises as a result of difficulties to identify seasonal patterns clearly for other diseases.

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Appendix

Time Series Plots for Epidemiological Data

