Performance of Maximum Likelihood and Bayesian Estimators for the Poisson Distribution

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The Poisson distribution is the standard distribution to model count data having only one parameter, λ. To estimate λ, several point estimators are available in the literature and their performance could be varied under various situations. Among the several available estimators, the commonly used estimator is the maximum likelihood estimator (MLE). This work compares the performance of Bayesian estimators, based on gamma prior squared error loss (SEL) and quadratic loss (QL) functions, with MLE in various situations with respect to the mean square error (MSE) sense by formulating a simulation study. The gamma distribution is a more flexible lifetime distribution having shape and scale parameters and it is an informative conjugate prior for λ. Here various situations were formulated by varying the gamma distribution parameters, sample size and λ. Further, two real-world applications (i) the number of horse-kick deaths of Prussian military personnel and (ii) customers' arrival at an ice cream stand to buy ice cream, were considered in order to verify the simulation study results. From the simulation study and real-world application results, we could conclude that the simulation study results are consistent with the real-world application results and performance of different estimators are affected by various gamma distribution parameters, sample size and λ. Further, for a higher sample-sized data set ($n > 50$), the performance of all given estimators are the same. When we have smaller sample-sized data set, MLE is the best option for a data set with a considerably higher mean and otherwise, the Bayesian estimator is the best option.

Keywords: Poisson Distribution, Bayesian Estimation, Prior Distribution, Squared Error Loss, Quadratic Loss, Mean Square Error