

**DIMENSION OF JOINT KERNEL OF PURE  $p$ -ISOPAIRS IN TERMS OF UNITARIES REPRESENTING  $p$  FOR  $p(z, w) = z^n - w^2$**

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An inner toral polynomial is a polynomial in two complex variables whose zero set is contained in  $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$ , where  $\mathbb{D}$ ,  $\mathbb{T}$  and  $\mathbb{E}$  represent the open unit disk, the unit circle and the exterior of the closed unit disk, respectively. Given an inner toral polynomial, there exists a minimal version  $p$  where  $p$  is minimal in the sense that  $p$  divides any polynomial with the same zero set as itself. A zero of a polynomial  $p(z, w)$  is called a regular point for  $p$ , if  $p$  has a non-zero gradient evaluation. For a minimal inner toral polynomial  $p(z, w)$  of bidegree  $(n, m)$ , there exists a unitary matrix  $U = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$  of size  $(m+n) \times (m+n)$  representing  $p$  such that  $p(z, w)$  is a constant multiple of  $\det \begin{pmatrix} A - wI & zB \\ C & zD - I \end{pmatrix}$ . A pure isometry is an isometry defined on a Hilbert space that behaves like a shift operator. Given a pair of pure isometries  $(S, T)$ , its bimultiplicity is defined as  $(\dim(\ker(S^*)), \dim(\ker(T^*)))$ , where  $*$  denotes the adjoint operator. A pair of pure isometries  $(S, T)$  satisfying the algebraic relationship  $p(S, T) = 0$ , where  $p$  is a polynomial, is called a pure  $p$ -isopair. It was proven that, for a fixed minimal inner toral polynomial  $p$  with bidegree  $(n, m)$ , and a unitary matrix  $U$  representing  $p$ , there exists a Hilbert space  $H$  such that the pair  $(M_z, M_w)$  on  $H$  is a pure  $p$ -isopair. Further, for a pure  $p$ -isopair  $(S, T)$  with finite bimultiplicity  $(m, n)$ ,  $\dim[\ker(S - \lambda I)^* \cap \ker(T - \mu I)^*] = 1$  whenever  $(\lambda, \mu) \in \mathbb{D}^2$  is a regular point for  $p$ . It can be observed that at a non-regular point  $(\lambda, \mu)$ , the above dimension of the joint kernel could be one or higher. In this work, we proved that for minimal inner toral polynomials of the form  $p(z, w) = z^n - w^2$  where  $n \geq 2$ , if the unitary matrix representing  $p$  has a zero diagonal and a non-zero block matrix  $A$ , then there exists a pure  $p$ -isopair  $(S, T)$  such that dimension of  $\ker((S - \lambda I)^*) \cap \ker((T - \mu I)^*)$  at the non-regular point  $(0, 0)$  is one.

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