

**SPLITTING HOMOGENEOUS POLYNOMIALS OVER
QUOTIENTS OF GROUP RINGS**

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This study investigates the possibility of factoring homogeneous multivariable polynomials into linear factors over quotients of group rings. The motivation for this research stems from the observation that group determinants are homogeneous multivariable polynomials, and they can be factored into linear factors over a quotient of real group rings associated with the relevant group. This raises the natural question of whether this result can be extended to arbitrary homogeneous multivariable polynomials. It can be easily proven that any given homogeneous multivariable polynomial can factor into linear factors in some finitely generated real algebra, which may not necessarily be generated by its units. This study proved that a finitely generated real algebra is isomorphic to a quotient of a real group ring of some finitely generated group if and only if it can be generated by its units. The first isomorphism theorem, and the universal property of group rings are the main tools in proving this theorem. Based on some direct computations, it was conjectured that for a given homogeneous multivariable polynomial there exists a finitely generated real algebra generated by its units, in which the polynomial splits into linear factors.

Keywords: Finitely generated real algebras, Group rings, Homogeneous multivariable polynomials