Modeling the Structural Relationship between the Underemployment and Shadow Economy

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INTRODUCTION

The degree of impact from underutilization of the labor force to the growth of the shadow economy can vary according to the type of economy. Within underdeveloped economies, the majority who employed are not usually having a wage that can fulfill their needs. Even due to the job-skill mismatch, people with higher educational qualifications are also underutilized. This provides incentives for them to engage in the shadow economy (Samaranayake & Dayaratne-Banda, 2015; Balkouti & Boujelbne, 2018).

This study attempts to develop a structural relationship between the rate of underemployment and the shadow economy. Accordingly, the theoretical derivation was led by two landmark research done by Hall (1979) and Dell'Anno & Solomon (2008). Hall (1979) focused on the duration of employment as long as both employers and employees care about it. Dell'Anno & Solomon (2008) had derived a structural relationship between the shadow economy and the rate of unemployment by adopting the Hall's (1979) model. Accordingly, this study intends to extend the development by Dell'Anno & Solomon (2008) and introduced "the rate of underemployment" as a new-fangled proxy.

RESEARCH OBJECTIVE

The underemployment appears to be one of the emerging factors in the underdeveloped nations which determine the degree of public enrollment in the shadow economy (Samaranayake & Dayaratne-Banda, 2015; Balkouti & Boujelbne, 2018). Though the literature had derived this idea throughout the empirical research, proper theoretical explanation is required to utilize the data on underemployment to estimate the magnitude of the shadow economy.

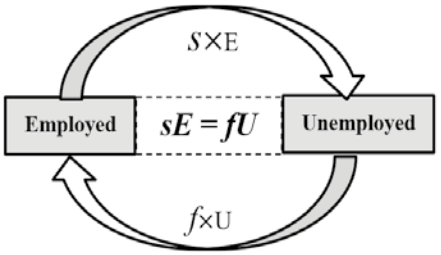
The foremost objective of this study is to develop a structural relationship between the rate of underemployment and the magnitude of the shadow economy. It is supposed to adopt the model

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by Hall (1979) and the development by Dell'Anno & Solomon (2008) in order to derive the proposed theoretical relationship.

METHODS

The theoretical investigation focuses on the model developed by Hall (1979) at the forefront of the proposed structural development. The analysis consists of a partial analysis for both job separation (s) and job finding (f) rates.



Therefore, the transitions between employment and unemployment and the steady-state have been considered together with both rates above and perform certain modifications to the model developed by Dell'Anno & Solomon (2008) via utilizing "the rate of underemployment" as a new-fangled proxy.

RESULTS

This analysis concerns a situation with the presence of underemployment. Therefore, this study introduces the rate of underemployment "U^*" into the closed economic model that includes the objective of creating an efficient labor market to make the rate of underemployment (U^*) zero. If the total labor force of "L" workers is constant, the labor force consists of workers who are Employed(E), Unemployed (U) and Underemployed (U^*).

$$\boldsymbol{L = U + U^* + E} \qquad (1)$$

Then the rate of underemployment = $\boldsymbol{U^*/L}$

When the job separation rate (s) determines a rate where worker losses a job and becomes unemployed, the job-finding rate (f) determines a rate where a worker finds a job and becomes employed. However, with the presence of underemployment in an economy, job-finding rate should be determined with both proper employment and underemployment. Then if we consider the steady-state equilibrium without considering the impact from underemployment, it may lead to a situation which inflates the precise magnitude of the job-separation rate and expose the steady-state, $s = f$, Therefore, steady-state equilibrium can be written as follows.

At the steady state, (equilibrium) $\boldsymbol{s + f^{\wedge} = f}$ (2)

Here, the impairment of job separation rate *($f^{\wedge}$)* is added to the presence of underemployment. Here $f^{\wedge}$ is used as the notation, because the reason to underestimate the job separation rate is underemployment, a part which improves the job finding rate.

From the steady state $s = f - f^{\wedge}$

Then, $(f - f^{\wedge})U = sE$ (3)

When underemployment = U^*,

Underemployment is a part of total employment. If the percentage of proper employment ($\boldsymbol{E}^*$) is λ proportion of the total employment, then the underemployment can be recognized as follows:

$E - \lambda E^* = U^*$

Then, $\boldsymbol{E} = \boldsymbol{U}^* + \lambda \boldsymbol{E}^*$ (4)

From the equation (3) with the presence of underemployment,

$(f - f^{\wedge})U = sE$

Then, $E = \frac{(f-f^{\wedge})U}{s}$ (5)

When we substitute equation (4) to the above,

$U^* + \lambda E^* = \frac{fU-f^{\wedge}U}{s}$

$U^* = \frac{fU-f^{\wedge}U}{s} - \lambda E^*$ (6)

When $s = \frac{U}{L}(f + s)$ is derived from Dell'Anno & Solomon (2008) and adjusted to a situation with the presence of underemployment, it is,

$s = \frac{U}{L}(f - f^{\wedge} + s)$ (7)

Then substitute (7) to (6),

$U^* = \frac{fU - f^{\wedge}U}{U/L(f - f^{\wedge} + s)} - \lambda E^*$

$U^* = \frac{(f - f^{\wedge})L}{(f - f^{\wedge} + s)} - \lambda E^*$

$\frac{\boldsymbol{U}^*}{\boldsymbol{L}} = \frac{(\boldsymbol{f}-\boldsymbol{f}^{\wedge})}{(\boldsymbol{f}-\boldsymbol{f}^{\wedge}+\boldsymbol{s})} - \frac{\boldsymbol{\lambda E}^*}{\boldsymbol{L}}$ (8)

Therefore, a compound equation is derived from equation (8) for the rate of underemployment. Further, it clearly reveals that higher the magnitude of λ hindering the magnitude of the rate of underemployment and *vice versa*.

The next phase of structural development considers the characteristics of the shadow economy. If all goods in the official economy are produced using only labor, and labor is only the stock in employment, then the output of the official economy can be demonstrated as follows,

If the output of the official economy = Y_1

Function of Technology = A

Therefore, $Y_1 = AE$

If technology is assumed to be constant for the short run, then $Y_1 = E$

Since $L = U + E$ and $E = U^* + \lambda E^*$,

Then $L = U + U^* + \lambda E^*$ and $U^* = L - U - \lambda E^*$

$$\frac{U^*}{L} = \frac{L - U - \lambda E^*}{L}$$

When $U = L - E$

$$\frac{U^*}{L} = \frac{L - (L - E) - \lambda E^*}{L}$$

If the whole stock of workers is engaged in producing the official output of the economy,

Then $Y_1 = E$,

$$\frac{U^*}{L} = \frac{L-(L-Y_i)-\lambda E^*}{L}$$

$$\boldsymbol{\frac{U^*}{L} = \frac{Y_1-\lambda E^*}{L}} \qquad (9)$$

According to the way Dell'Anno & Solomon (2008) we considered the total output = Y_T, the official output = Y_1 and the output produced in shadow economy = Y_2.

Therefore, $Y_T = Y_1 + Y_2$ and then,

$$\boldsymbol{Y_1 = Y_T - Y_2} \qquad (10)$$

Substitute (10) to (9),

$$\frac{U^*}{L} = \frac{(Y_T - Y_2) - \lambda E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{U^*}{L} - \frac{\lambda E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \left(\frac{U^*}{L} + \frac{\lambda E^*}{L}\right) \quad (11)$$

Then substitute (8) to (11)

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \left[\frac{(f-f^\wedge)}{(f-f^\wedge+s)} - \frac{\lambda E^*}{L}\right] - \frac{\lambda E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{(f-f^\wedge)}{(f-f^\wedge+s)}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{f}{(f-f^\wedge+s)} + \frac{f^\wedge}{(f-f^\wedge+s)} \quad (12)$$

The above equations (11) and (12) represent the nature of the relationship between the shadow economy, employment and the rate of underemployment. As given in equation (11), the shadow economy demonstrates a negative relationship with the rate of employment. Employment may consist of both underemployed and properly employed workers. However, further derivations from equation (12) provided that the negative relationship will not be a common factor for each and every economy.

DISCUSSION

The study uses a series of equations to investigate possibilities of relationship exists between underemployment and the size of the shadow economy. Ultimately, the derivation is given a possibility to have either a positive or negative relationship between two variables as follows.

$$\therefore \frac{Y_2}{L} = \frac{Y_T}{L} - \frac{f}{(f-f^\wedge+s)} + \frac{f^\wedge}{(f-f^\wedge+s)}$$

The magnitude of '$f^\wedge$' can introduce as the determinant which decide the nature of relationship. With the absence of $f^\wedge$ *(if* $f^\wedge$*=0)* there is usual negative relationship between employment and shadow economy. On the other hand, higher the magnitude of $f^\wedge$ will turn this relationship in to a positive where $f = f^\wedge$ is the critical condition which determine the turnover of the existence of either positive or negative relationship.

KEYWORDS

Shadow Economy, Unemployment, Underemployment, Job-Finding Rate, Job-Separation Rate

REFERENCES

Baklouti, N. & Boujelbene, Y (2018). 'A simultaneous equation model of economic growth and shadow economy: Is there a difference between the developed and developing countries?'. *Economic Change and Restructuring*. pp. 1-20. Available at: https://doi.org/10.1007/s10644-018-9235-8.

Dell'Anno, R & Solomon, O.H. (2008). 'Shadow economy and unemploy ment rate in USA: is there a structural relationship? An empirical analysis'. *Applied Economics*. Vol.40, pp.2537-2555.

Hall, R.H. (1979). 'A Theory of the Natural Unemployment Rate and the Duration of Employment'. *Journal of Monetary Economics*. Vol.5, pp.153-169.

Samaranayake, D.I.J. & Dayarathna-Banda, O.G. (2015). 'Unemployment, Official Economy and the Dimension of the Shadow Economy; an Empirical Analysis for Sri Lanka Using SEM Approach'. *Sri Lanka Journal of Economic Research*. Vol.3, no.1, pp 113-153.