***Abstract No: 200*** ***ICT, Mathematics, and Statistics***

# NEW ITERATIVE TECHNIQUE FOR SOLVING NONLINEAR EQUATIONS

**H.H. Safwathuz Ziya*, W.A. Gunarathna and M.A.M. Mohamed**

*Department of Physical Science, Rajarata University of Sri Lanka, Mihinthale, Sri Lanka*
**safwahasan319@gmail.com*

Finding roots of Non-linear Equations (NEs) in the form $f(x) = 0$ over $\mathbb{R}$ is a common problem in many engineering and scientific applications. Direct methods to find the roots of NEs are limited. For example, while direct formulas exist for polynomial equations up to the fourth degree, no such methods are available for transcendental equations. Consequently, iterative methods for searching roots of NEs have long been considered for research. The Bisection method, Regula-Falsi Method (RFM), Secant Method (SM), and Newton-Raphson Method (NM) are well-known classical iterative techniques. The rate of convergence, or convergence order ($p$) of an iterative technique is of great importance since it measures how fast it can approach the root of the NE. It is known that the rate of convergence of the RFM is linear. This study aims to construct an iterative technique with a superior convergence order compared to the RFM. This study proposed a new iterative technique for searching simple roots of non-linear equations. To construct this technique, a convex combination of the Newton-Raphson formula at two points is considered so that its iterative formula is given by: $\text{NewM}_\lambda$: $x_{n+1} = \lambda\left(x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}\right) + (1-\lambda)\left(x_n - \frac{f(x_n)}{f'(x_n)}\right), \quad 0 \le \lambda \le 1,\ n = 1, 2, 3, \ldots$. The convergence analysis of the proposed method is also established. It displays a superlinear convergence over RFM, with order $p = \sqrt{2}$. The performances of $\text{NewM}_\lambda$ are illustrated by several numerical examples, confirming its effectiveness in searching roots. For instance, with an accuracy of $|f(x)| \le 10^{-15}$, the root (number of iterations) of $f(x) = (x+2)e^x - 1 = 0$ in $(-1, 1)$ is obtained by $\text{NewM}_{0.5}$, RFM, SM, and NM are $-0.4428544010023885(12)$, $-0.4428544010023892(85)$, $-0.4428544010023886(10)$, and $-0.4428544010023885(6)$, respectively.

**Keywords:** Convergence order, Newton method, Non-linear equations, Simple roots