

COMPARISON OF RADIAL BASIS FUNCTION GLOBAL METHOD AND RADIAL BASIS FUNCTION-FINITE DIFFERENCE (LOCAL) METHOD AS AN INTERPOLATION TECHNIQUE

W.H.D.T. Karunarathna^{1, 2*}, K.M.N. Dissanayake³ and M.T.M. Dewasurendra¹

¹Department of Mathematics, University of Peradeniya, Peradeniya, Sri Lanka

²Postgraduate Institute of Science, University of Peradeniya, Peradeniya, Sri Lanka

³Department of Mathematical Sciences, Appalachian State University, Boone, North Carolina, USA

*dulashinikarunarathna111@gmail.com

The Radial Basis Function (RBF) method is a numerical method that can be used to solve interpolation problems and partial differential equations (PDEs). There are two main types of RBFs: infinitely smooth RBFs and piecewise-smooth RBFs. Gaussian, Multiquadric, Inverse Multiquadric, and Inverse Quadric functions are examples of infinitely smooth RBFs commonly used with the global RBF approach. An important factor of the infinitely smooth RBFs is that they contain a shape parameter. When using the global RBF method, one has to choose a suitable value for the shape parameter as it dramatically impacts convergence. In addition, using the global RBF method is computationally expensive as it produces dense matrices. Therefore, the possibility of using the local RBF approach, known as the RBF-FD approach, is studied, along with shape parameter independent Polyharmonic Spline RBFs, to overcome the aforementioned obstacles. In order to create RBF-FD interpolation stencils, Polyharmonic Spline stencils augmented with polynomials were used. This is a common approach used to solve PDEs, which we adapt to solve interpolation problems. In this work, we interpolated $f(x, y) = xe^{-x^2-y^2}$ with various known nodes and fixed 6400 unknown 2D nodes on $[-1, 1]$ by using Gaussian, Multiquadric and Polyharmonic Spline RBFs. Also, we calculated the error of the approximation for different known numbers of 2D nodes. The accuracy of the solution oscillated with the shape parameter. However, when we used the RBF-FD method, we observed a clear pattern of error decay when increasing the number of nodes. The order of convergence was in the realm of at least $O(h^p)$ to a maximum of $O(h^{p+2})$ where h is the fill distance and p is the degree of the appended polynomial. In addition, unlike the global approach, the RBF-FD method produced sparse matrices, which leads to a computationally efficient and stable algorithm.

Keywords: Interpolation method, Radial Basis Function method, Shape parameter