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SYMBOLIC AND ALGORITHMIC INSIGHTS INTO RAINBOW VERTEX ANTIMAGIC COLORING OF WHEEL GRAPHS

D.M.N.U.B. Dasanayake* **and P.G.R.S. Ranasinghe**

Department of Mathematics, University of Peradeniya, Peradeniya, Sri Lanka.
*nimeshikabandaradasanayake@gmail.com

This study investigates the Rainbow Vertex Antimagic Coloring (RVAC) - a hybrid of antimagic labeling and vertex coloring of wheel graphs W_n using a combination of symbolic formulas and algorithmic computation. In RVAC, edges are uniquely labeled so that each vertex's color, given by the sum of its incident edges, is distinct while minimising the total number of colors. Formally, for a simple, connected, undirected graph $G = (V, E)$ with a labeling function $f: E(G) \rightarrow \{1,2, \dots, | E(G) |\}$, the weight of a vertex $v \in V(G)$ is defined as $w_f(v) = \sum_{e \in E(V)} f(e)$, where $E(v)$ is the set of edges incident to v. The function f is a vertex antimagic edge labeling if all vertex weights are distinct. A rainbow path is a path whose internal vertices have distinct weights, and the rainbow vertex antimagic connection number $rvac(G)$ is the minimum number of colors needed so that every pair of vertices is connected by a rainbow path. Unlike traditional antimagic labeling or rainbow connections, RVAC explicitly integrates both vertex color distinctness and edge-sum constraints. For wheel graphs W_n, it has been observed that that $rvac(Wn) = 3$ when n is odd and $rvac(Wn) = 2$ when n is even. During analysis, counterexamples arose when applying direct edge labeling, highlighting the limitations of this approach. To address this, a general parity-aware labeling scheme was developed that systematically assigns vertex weights and determines the RVAC directly from these weights, eliminating the need for explicit edge enumeration. A Python-based implementation automates graph construction, applies the labeling formulas, verifies RVAC conditions, and generates visual representations, enabling efficient validation for large n (up to 10^4). This symbolic–algorithmic approach offers a scalable framework for studying RVAC in wheel graphs and other graph families, demonstrating the integration of theoretical reasoning with algorithmic automation for broader mathematical and applied use, with applications in combinatorics, cryptography, and network security.

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