

**GENERALIZATION OF THE SYMMETRICITY PROPERTIES OF
POLYNOMIALS DEFINING DISTINGUISHED VARIETIES ON THE OPEN UNIT
DISK**

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Let \mathbb{D} be the open unit disk, \mathbb{T} be the unit circle, and \mathbb{E} be the exterior of the closed unit disk in \mathbb{C} . A polynomial $p(z, w)$ is said to define a distinguished variety on \mathbb{D}^2 , if $Z(p) = \{ (z, w) \in \mathbb{C}^2 : p(z, w) = 0 \} \subseteq \mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$. For such polynomials, the zero set $Z(p)$ inside \mathbb{D}^2 is called a distinguished variety on \mathbb{D}^2 . For a polynomial $p(z, w)$ with two variables having bidegree (n, m) , the reflection is defined by $\tilde{p}(z, w) = z^n w^m \overline{p\left(\frac{1}{z}, \frac{1}{w}\right)}$. A polynomial $p \in \mathbb{C}[z, w]$ is essentially \mathbb{T}^2 -symmetric if $p(z, w) = c\tilde{p}(z, w)$ for some $c \in \mathbb{T}$. In 2010, Greg Knese introduced the concept of symmetricity for polynomials defining distinguished varieties on \mathbb{D}^2 and has shown that a polynomial p defining a distinguished variety on \mathbb{D}^2 is essentially \mathbb{T}^2 -symmetric. In this study, this concept of symmetricity is generalized for polynomials defining distinguished varieties on open unit polydisk \mathbb{D}^n , by considering polynomials with n variables. A polynomial $p(z_1, z_2, \dots, z_n)$ is said to define a distinguished variety on \mathbb{D}^n , if $Z(p) = \{ (z_1, z_2, \dots, z_n) \in \mathbb{C}^n : p(z_1, z_2, \dots, z_n) = 0 \} \subseteq \mathbb{D}^n \cup \mathbb{T}^n \cup \mathbb{E}^n$. For such polynomials, the zero set $Z(p)$ inside \mathbb{D}^n is called a distinguished variety on \mathbb{D}^n . For a polynomial $p(z_1, z_2, \dots, z_n)$ with n variables having degree (m_1, m_2, \dots, m_n) , the reflection is introduced by $\tilde{p}(z_1, z_2, \dots, z_n) = z_1^{m_1} z_2^{m_2} \dots z_n^{m_n} \overline{p\left(\frac{1}{z_1}, \frac{1}{z_2}, \dots, \frac{1}{z_n}\right)}$. We defined a polynomial $p \in \mathbb{C}[z_1, z_2, \dots, z_n]$ to be essentially \mathbb{T}^n -symmetric if $p(z_1, z_2, \dots, z_n) = c\tilde{p}(z_1, z_2, \dots, z_n)$ for some $c \in \mathbb{T}$. This study proves that a polynomial p defining a distinguished variety on \mathbb{D}^n is essentially \mathbb{T}^n -symmetric for any $2 \leq n < \infty$. Future studies can focus on proving properties that already exist for two variable polynomials in the case of polynomials with n variables.

Keywords: Distinguished varieties, Inner toral polynomials, Symmetricity