Structural Relationship between Under-Employment and Shadow Economy: A Theoretical Investigation

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Introduction

Shadow economy reflects a part of economic activities involving goods and services which are paid by cash but not declared for tax. The affiliation between shadow economy and labour force is vastly discussed upon the relationship with the rate of unemployment and shadow economy (Dell'Anno and Solomon, 2008, Schneider et.al, 2010, Davidescu and Dobre, 2012). However, as mentioned by Sengenberger (2011) "*unemployment statistics are often criticized in developed and in developing countries alike - for failing to reflect the 'true' extent of the employment problem*", unemployment seems not a border challenge for emerging economies when compared to new challenges occur with underutilization and inefficiency matters of contemporary labor market.

The degree of impact from underutilization of labour force to the growth of shadow economy can vary according to the type of economy. Within under developed economies, majority of people who employed are not usually having a wage rate which can fulfill their all needs. Therefore, this may worse with under-employed people. In addition to that people may underemployed within such economies due to lack of job opportunities and vocational skills. Then people with higher educational qualifications also engaged with low grade jobs with lower

salaries. This may provide incentives for them to engage in shadow economic activities to earn more. As a result of that study attempts to develop a structural relationship between under-employment and shadow economy. Under-employment reflects a situation where employee works, but not belongs to their actual or exact capacity. Here this 'capacity' will identify in different prospects such as underutilization, underestimate or undermine the employee skills and capabilities LFS-Annual report (2014); Walling and Clancy (2010); ADB (2005).

Objective

This study attempts to develop a structural relationship between the rate of under-employment and shadow economy.

Methodology

The study uses the structural relationship developed by Dell'Anno and Solomon (2008) between the level of unemployment and shadow economy. Study also uses certain assumptions in advance to facilitate the core of the theoretical argument and its rationality in a situation with the presence of under-employment issue at the economy.

Results and discussion

Consider a closed economy model that includes objective of an efficient labor market to let the under-employment ($\mathbf{U}^{*}$) to zero. This analysis may concern a situation with the presence of under-employment at the economy.

Let's assume that the total labour force is L workers and it is constant. The labour force consist stock of workers who are employed (E) and Unemployed (U).

$$L = U + E$$

Then the rate of unemployment = U/L

When the job separation rate (s) determines a rate where employee losses a job and becomes unemployed, the job finding rate (s) determines a rate where a person find a job and become employed. However, with the presence of under-employment in the economy, job finding rate should be determined with both proper employment and under-employment. Then if we concern the steady state equilibria without considering the impact from under-employment, it may give a situation where s<f. This means that the presence of under-employment will devaluate the job separation rate. Because, workers who under-employed does not prefer for their current job but stay at the same place due to lack of proper and suitable job opportunity for his/her skills and preference. Therefore steady state equilibria can be written as below by using impact from both proper employment and under-employment to determine 'f'.

At the steady state, (equilibria): $s + f^* = f$

Where add the damage of job separation rate (f^*) with the presence of under-employment. Here 'f^*'use as the notation, because the reason for underestimate the job preparation rate is under-employment, a part which increase the job preparation rate. When we are considering the stock of workers who are employed (E) and unemployed (U) under these rates,

From the steady state: $s = f - f^*$

Then, $(f - f^*)U = sE$ (1)

When under-employment = U^*,

Underemployment is a part of total employment. If the percentage of proper employment (E^* – employees who are not under-employed) is

60 percent of total employment[2], then the under-employment can be recognize from the equation below.

$$E - 0.6E^* = U^*$$

Then, $E = U^* + 0.6E^*$ (2)

From the equation (1) with the presence of under-employment

$$(f - f^*)U = sE$$

Then, $E = \frac{(f-f^*)U}{s}$

When substitute equation (2) to above,

$$U^* + 0.6E^* = \frac{fU-f^*U}{s}$$

$$U^* = \frac{fU-f^*U}{s} - 0.6E^* \quad (3)$$

When $\frac{U}{L} = \frac{s}{(f+s)}$ is the equation derived by Dell'Anno and Solomon (2008) for the relationship exists between unemployment rate with both 's' and 'f'.

In a situation with the presence of under-employment,

$$s = \frac{U}{L}(f - f^* + s) \quad (4)$$

Then substitute (4) to (3),

$$U^* = \frac{fU - f^*U}{U/L\,(f - f^* + s)} - 0.6E^*$$

$$U^* = \frac{(f - f^*)L}{(f - f^* + s)} - 0.6E^*$$

$$\frac{U^*}{L} = \frac{(f-f^*)}{(f-f^*+s)} - \frac{0.6E^*}{L} \quad (5)$$

[2] This is an assumption that 60 percent of employees from total employees are perfectly employed and the rest of 40 percent are under-employed.

If all goods in official economy are produced using only labour and labour is the only stock that are in employment, then output of official economy can be exhibited as below.

If the output in the official economy = Y_1

Function of technology = A

Stock of workers who are employed = E

Therefore, $Y_1 = AE$

If technology assumed to be constant, then $\mathbf{Y_1 = E}$

Since $L = U + E$, then $L = U + U^* + 0.6E^*$

$$U^* = L - U - 0.6E^*$$

$$\frac{U^*}{L} = \frac{L - U - 0.6E^*}{L}$$

When $U = L - E$

$$\frac{U^*}{L} = \frac{L - (L - E) - 0.6E^*}{L}$$

When $Y_i = E$,

$$\frac{U^*}{L} = \frac{L-(L-Y_i)-0.6E^*}{L}$$

$$\frac{U^*}{L} = \frac{Y_1-0.6E^*}{L} \qquad (6)$$

When the total output = Y_T and the output produced in shadow economy = Y_2

$Y_T = Y_1 + Y_2$ and then,

$$Y_1 = Y_T - Y_2 \qquad (7)$$

Substitute (7) to (6)

$$\frac{U^*}{L} = \frac{(Y_T - Y_2) - 0.6E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T - 0.6E^*}{L} - \frac{U^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{U^*}{L} - \frac{0.6E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \left(\frac{U^*}{L} + \frac{0.6E^*}{L}\right) \quad (8)$$

Then substitute (5) to (8)

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \left(\frac{(f-f^*)}{(f-f^*+s)} - \frac{0.6E^*}{L}\right) - \frac{0.6E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{(f-f^*)}{(f-f^*+s)} + \frac{0.6E^*}{L} - \frac{0.6E^*}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{(f-f^*)}{(f-f^*+s)}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{f}{(f-f^*+s)} + \frac{f^*}{(f-f^*+s)} \quad (9)$$

Above equations 8 and 9 are represent the nature of relationship between shadow economy, the rate of employment and the rate of under-employment. As in equation (8), shadow economy demonstrates a negative relationship with the rate of employment. Employment may consist both under-employed and properly employed workers. However, further derivations from equation (9) give further detail on this relationship. Above negative relationship will not a common factor for each and every economy. Increase in the stock of workers who find jobs (f) in formal sector increase the employment rate and this has resulted in a decline in shadow economy as a proportion of labour force. Increase in stock of workers who are not properly employed (f*) will improve the size of shadow economy. Moreover, the rate of proper employment (f - f*) and job separation rates may determine the size of both total and underutilized job finding rates. When f* = 0, there will be perfect negative relationship between under-employment and the

size of shadow economy. Moreover, the validity of above outcome can be checked through re arranging the outcome to observe the nature of relationship between shadow economy, the rate of employment and the rate of unemployment.

When $E = U^* + 0.6E^*$, then $E^* = \frac{E-U^*}{0.6}$

Substitute above equation for E^* to equation 6.

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{U^*}{L} - \frac{0.6\left(\frac{E-U^*}{0.6}\right)}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{U^*}{L} - \frac{(E-U^*)}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{U^*}{L} + \frac{U^*}{L} - \frac{E}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{E}{L} \qquad (10)$$

According to equation 10, there is an inverse relationship between shadow economy and level of employment.

On the other hand, since $E = L - U$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{(L-U)}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} - \frac{L}{L} + \frac{U}{L}$$

$$\frac{Y_2}{L} = \frac{Y_T}{L} + \frac{U-L}{L} \qquad (11)$$

Equation 11 demonstrates the positive relationship between shadow economy and unemployment. Therefore the outcome derive an inverse relationship between shadow economy and under-employment rate is presence according to the previous theoretical derivations and may be useful to understand the nature of relationship between under-employment and the shadow economy to some extent.

Conclusion

Study use series of equations to investigate possibilities of relationship exists between the rate of under-employment and the size of shadow economy. Findings of these equations are given a possibility to have either positive or negative relationship between the rate of under employment and the shadow economy.

The magnitude of damage occurred within the job separation rate (f*) can introduce as the determinant which decide the nature of relationship. With the absence of f* (if f*=0) there is usual negative relationship between the rate of under-employment and shadow economy. On the other hand, higher the magnitude of f* will turn this relationship into a positive where $f = f^*$ is the critical condition which determine the turnover of the existence of either positive or negative relationship.

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