

RESULTS ON VARIOUS CLOSED SETS IN BI-GENERALISED TOPOLOGICAL SPACES

M. Arunmaran^{*}, P. Sachivathanan and K. Kannan

Department of Mathematics and Statistics, University of Jaffna, Jaffna, Sri Lanka
^{*}marunmaran03@gmail.com

The exploration and examination of bi-generalized topological spaces are driven by the aim of investigating wider generalizations of topological spaces, which facilitates a more adaptable framework for analyzing topological properties and their interrelations. The concept of bi-generalised topological space was introduced by Boonpok in 2010. The objective of this study is to provide some new results in the sets, namely (m, n) -closed sets and (m, n) -generalised closed sets in bi-generalised topological spaces. In addition, two new types of closed sets are defined: (m, n) - α^* -closed sets and $(m, n) - \beta^*$ -closed sets, and some results for these new closed sets are provided. For a non-empty set X , the triple (X, μ_1, μ_2) is a bi-generalised topological space, where μ_1 and μ_2 are generalised topologies on X . The members of μ_m are called μ_m -open sets, $m = 1, 2$. The complement of μ_m -open set is μ_m -closed set. A subset A of a bi-generalised topological space (X, μ_1, μ_2) is called (m, n) -closed if $cl_{\mu_m}(cl_{\mu_n}(A)) = A$, where $m, n = 1, 2$ with $m \neq n$. Similarly, the set A in X is called (m, n) -generalised closed if $cl_{\mu_n}(A) \subseteq U$, whenever $A \subseteq U$ and U is μ_m -open set in X . In this study, firstly, it is shown that if the intersection or union of two subsets of a bi-generalised topological space is (m, n) -closed, then the subsets need not be (m, n) -closed. Next, a similar result for (m, n) -generalised closed sets are shown. Secondly, $(m, n)\alpha^*$ -closed set is defined as follows: In a bi-generalised topological space (X, μ_1, μ_2) , a subset A in X is called $(m, n)\alpha^*$ -closed if $int_{\mu_m}(cl_{\mu_n}(A)) \subseteq U$ for every $A \subseteq U$ and U is an (m, n) -open set. Then, it is shown that the intersection of two $(m, n)\alpha^*$ closed sets need not be $(m, n)\alpha^*$ closed. Also, if the intersection of two sets A and B is $(m, n)\alpha^*$ closed, then the sets A, B need not be $(m, n)\alpha^*$ closed. Similarly, it can be shown that the union of two $(m, n)\alpha^*$ closed sets need not be $(m, n)\alpha^*$ closed. Also, it is proved that a subset A in a bi-generalised topological space (X, μ_1, μ_2) is $(m, n)\alpha^*$ closed if and only if $X \setminus A$ is $(m, n)\alpha^*$ open. Finally, another closed set is introduced, namely $(m, n)\beta^*$ closed as follows: A set A in a bi-generalised topological space is called $(m, n)\beta^*$ closed if $cl_{\mu_n}(int_{\mu_m}(A)) \subseteq U$ for every A contained in U , where U is (m, n) -open set. Also, this study found that the intersection or union of two $(m, n)\beta^*$ closed sets need not be $(m, n)\beta^*$ closed.

Keywords: Bi-generalised topology, $(m, n)\alpha^*$ closed, $(m, n) - \beta^*$ closed, (m, n) -closed set, (m, n) -generalised closed set