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DOMINATING COMMON FACTOR METRIC ON RATIONAL NUMBERS

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This study introduces a novel metric on the set of positive rational numbers, derived by pulling back the symmetric difference metric on finite sets along a generalized divisor function. We first generalize the classical notion of divisibility to rational numbers via a prime factorization-based criterion. Using this definition, each rational number is associated with a finite set of its divisors, analogous to the classical set of divisors for positive integers. Subsequently, a distance function on positive rationals is defined by $d(r,s) = \sigma_0(r) + \sigma_0(s) - 2\sigma_0\big(\mathrm{dcf}(r,s)\big)$, where $\sigma_0(r)$ denotes the number of divisors of r and $\mathrm{dcf}(r,s)$ is a generalized greatest common divisor. The study establishes a characterization of minimal distances and nearest neighbours in the induced metric space, reflecting the arithmetic-geometric structure of numbers in it. It was proved that, given a positive rational number, its nearest neighbours are precisely those obtained by increasing or decreasing the exponent of one of its most frequent prime factors by one. A complete description of unit spheres was provided: two positive rational numbers are at unit distance if and only if they are of the form p^r and p^{r+1} for some $r \in \mathbb{Z}$ and some $p \in \mathbb{P}$. These results contribute to a broader understanding of how multiplicative arithmetic properties influence interactions between rational numbers and open avenues for applications of metric space theory in multiplicative number theory.

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