GROUP OBJECTS IN THE CATEGORY OF SIMPLE REFLEXIVE GRAPHS

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Category theory is the study of mathematical structures and the relationships between them, taking objects and morphisms as fundamental notions, as opposed to the elements and member relationships of the set theory. Despite being introduced only about 80 years ago, category theory has become the main language of contemporary mathematics. As a result of category theory providing a conceptual unification to different abstract mathematical disciplines, one of its branches (categorical algebra) studies finitary universal algebraic structures, such as groups, defined within various categories. If a category has all finite products, one can form these generalized groups, known as group objects internal to the ambient category, on its objects which are potentially more complicated than sets. Topological groups, Lie groups, algebraic groups, simplicial groups, localic groups, cogroups, and commutative Hopf algebras are a few well-known examples of group objects in various categories. In fact, abelian groups (in the category of groups) and finitely generated free groups (in the opposite category of groups) are also two special types of group objects. The goal of this project is to identify the group objects internal to a particular category of graphs, which we call simple reflexive graphs. Compared to other categories of graphs, this category has exceptional categorical properties. Our main result is a theorem that characterizes group objects in this category: H is a group object in simple reflexive graphs if and only if there exists a group structure on $V(H)$ such that $E(H)$ is a subgroup of $V(H) \times V(H)$, where $V(H)$ and $E(H)$ are the set of vertices and the set of edges of H, respectively. In this case, H is a disjoint union of complete reflexive subgraphs, and connected components are isomorphic to each other.

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