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SYMMETRICITY OF POLYNOMIALS DEFINING DISTINGUISHED VARIETIES ON THE SYMMETRIZED BIDISK

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Let $\mathbb{D}$ be the unit disk, $\mathbb{T}$ be the unit circle, and $\mathbb{E}$ be the set $\mathbb{C} \setminus \overline{\mathbb{D}}$ in $\mathbb{C}$. For a polynomial $f(z,w) \in \mathbb{C}[z, w]$ such that its zero set $Z(f) \subset \mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, we say $Z(f) \cap \mathbb{D}^2$ is a distinguished variety on $\mathbb{D}^2$ and the polynomial $f(z,w)$ is called a polynomial defining a distinguished variety on $\mathbb{D}^2$. A polynomial $f(z,w) \in \mathbb{C}[z,w]$ is said to have bidegree (n,m) if $f(z,w)$ has degree n in z and degree m in w. The reflection of $f(z,w)$ at bidegree (n,m) is defined as the $\tilde{f}(z,w) = z^n w^m \overline{f\left(\frac{1}{\bar{z}}, \frac{1}{\bar{w}}\right)}$. A polynomial $f(z,w)$ is called essentially $\mathbb{T}^2 -$ symmetric if $f(z,w) = c\,\tilde{f}(z,w)$ for some $c \in \mathbb{T}$. Every polynomial that defines a distinguished variety on $\mathbb{D}^2$ is symmetric with respect to $\mathbb{T}^2$ in the sense that it is essentially $\mathbb{T}^2 -$ symmetric. Let $\pi : \mathbb{C}^2 \to \mathbb{C}^2$ be the symmetrization map given by $\pi : (z,w) \mapsto (z + w, zw)$ and $\mathbb{G} = \pi(\mathbb{D}^2), \Gamma$ be the boundary of $\mathbb{G}$ and $\partial\Gamma = \pi(\mathbb{T}^2)$ be the distinguished boundary of $\mathbb{G}$, where $\mathbb{G}$ is called the symmetrized bidisk. For a polynomial $g(s,p) \in \mathbb{C}[s,p]$ such that its zero set $Z(g) \subset \mathbb{G} \cup b\Gamma \cup \mathbb{C} \setminus \Gamma$, we say $Z(g) \cap \mathbb{G}$ is a distinguished variety on the symmetrized bidisk $\mathbb{G}$, and $g(s,p)$ is referred to as a polynomial defining a distinguished variety on the symmetrized bidisk $\mathbb{G}$. In this work, symmetric properties of polynomial defining a distinguished variety on the symmetrized bidisk $\mathbb{G}$ were studied. Given a polynomial $g(s,p)$ of bidegree (k,l), the reflection of g was defined as $\hat{g}(s,p) = p^k \overline{g\left(\frac{1}{\bar{p}/\bar{s}}, \frac{1}{\bar{p}}\right)}$. A polynomial g is essentially $\partial\Gamma$-symmetric if $g(s,p) = c\,\hat{g}(s,p)$, where $c \in \mathbb{T}$. It was proved that a polynomial defining a distinguished variety on the symmetrized bidisk is essentially $\partial\Gamma$- symmetric. This study contributes to the broader understanding of geometric representation and properties of polynomial defining distinguished varieties on the symmetrized bidisk.

Keywords: Distinguished varieties, Inner toral polynomials, Symmetrized bidisk