

PRODUCT CORDIAL LABELLING OF GENERALIZED HELM GRAPHS

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Graph labelling is the process of assigning integers to the vertices, edges, or both, under specific constraints. In 1987, Ibrahim Cahit introduced cordial labelling as a more flexible alternative to graceful and harmonious labellings. A specialized form, product cordial labelling, was later introduced by R. Ponraj, M. Sivakumar, and M. Sundaram in 2012, leading to further research on different classes of graphs that exhibit this property. Product cordial labelling is defined as a function $f: V(G) \rightarrow \{0,1\}$, where each edge uv is assigned the label $f(u)f(v)$. A graph is said to be product cordial if the absolute difference between the number of vertices labeled 0 and 1 is at most 1, and the absolute difference between the number of edges labeled 0 and 1 does not exceed 1. This labelling method provides insights into graph structures and their combinatorial properties, making it a useful tool in network theory, coding, and other applied fields. In this study, the existence of product cordial labelling for helm graphs H_n , obtained by attaching a pendant edge to each vertex of the cycle in an n -wheel graph, is shown. A structured approach is presented for labelling these graphs based on whether the cycle has an odd or even number of vertices. Furthermore, this study extends the analysis to a generalized version of helm graphs, such as those with additional pendant attachments, and explores conditions under which these graphs maintain product cordiality. These findings contribute to the broader study of graph labellings and their applications in discrete mathematics and theoretical computer science.

Keywords: Cordial labelling, Helm graph, Product cordial labelling, Wheel graph