Hadamard Graphs from Generalized Hadamard Matrices of Order p^n; $p \geq 3$ over a Cyclic Group C_p

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Hadamard matrices and their applications have steadily and rapidly grown during the last two decades. This research is focused on generalized Hadamard matrices which have applications in combinatorics such as graph theory and transversal design. In our work, a mathematical method for constructing generalized Hadamard graphs corresponding to the generalized Hadamard matrix $GH(p^n, C_p)$, $n \in \mathbb{N}$ is introduced using the properties of Latin squares. Let C be a finite abelian group of order w. A $v \times v$ matrix $M = [m_{ij}]$ with entries from C where w divides v is a generalized Hadamard matrix denoted by $GH(w, v/w)$ over C if, for all $i \neq j$, the sequence of quotients $m_{ij}m_{jk}^{-1}, 1 \leq k \leq v,$ contains each element of C exactly v/w times. To construct the corresponding Latin square, cyclic shifting method is used, and the elements of the Latin square are the elements of the generalized Hadamard matrices of order p^n over a cyclic group C_p. Those Latin squares are then used to find the edges in the resulting graphs. The graph obtained in this way has $2p^{n+1}$ vertices labelled $r_i^1, r_i^{\omega}, r_i^{\omega^2}, r_i^{\omega^3}, \dots, r_i^{\omega^p}, c_j^1, c_j^{\omega}, c_j^{\omega^2}, c_j^{\omega^3}, \dots, c_j^{\omega^p}$ where $i, j = 1, 2, \dots, p^n$. $p = 3$ case exists in the literature. The method was tested for $p = 5, n = 1$ and $n = 2$, and $p = 7, n = 1$ cases manually and the algorithm was automated. The resultant graphs for the examples are $5-$regular, $25-$ regular and $7-$regular respectively. The automated algorithm can be used to construct p^n-regular graphs.

Key words: Generalized Hadamard Graphs, Latin Squares, Cyclic Shifting