

**POLYNOMIALS DEFINING DISTINGUISHED VARIETIES IN A
GENERALISED VERSION OF SYMMETRISED BIDISC**

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In this study, we explored the characterisation of algebraic curves known as distinguished varieties that exhibit special boundary behavior of exiting the symmetrised bidisc exclusively through its distinguished boundary. Let \mathbb{D} be the open unit disc, \mathbb{T} be the unit circle, and \mathbb{E} be the $\mathbb{C} \setminus \overline{\mathbb{D}}$ in \mathbb{C} . Let $\mathbb{D}^2 = \mathbb{D} \times \mathbb{D}$ be the unit bidisc in \mathbb{C}^2 . For a polynomial $p(z, w) \in \mathbb{C}[z, w]$ such that its zero set $Z(p) \subseteq \mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, $Z(p) \cap \mathbb{D}^2$ is a distinguished variety in \mathbb{D}^2 . Let the symmetrisation map $\pi: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ be defined by $\pi(z, w) = (z + w, zw)$, and define the symmetrised bidisc $\mathbb{G} = \pi(\mathbb{D}^2)$, the distinguished boundary of \mathbb{G} be $b\Gamma = \pi(\mathbb{T}^2)$, and the exterior of \mathbb{G} be $\Omega = \pi(\mathbb{E}^2)$. For a polynomial $q(s, p) \in \mathbb{C}[s, p]$ such that its zero set $Z(q) \subseteq \mathbb{G} \cup b\Gamma \cup \Omega$, the set $Z(q) \cap \mathbb{G}$ is a distinguished variety in \mathbb{G} . It is proven that $W \subset \mathbb{G}$ is a distinguished variety in \mathbb{G} if and only if there exists a distinguished variety V in \mathbb{D}^2 such that $W = \pi(V)$. In this study, we partially generalised this result by considering a generalised version of symmetrised bidisc. Considering the map $\tilde{\pi}: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ given by $\tilde{\pi}(z, w) = (z + w, z^2 + w^2)$, let $\tilde{\mathbb{G}} = \tilde{\pi}(\mathbb{D}^2)$. By defining a distinguished variety in $\tilde{\mathbb{G}}$ in a similar fashion, we proved that $\tilde{W} \subset \tilde{\mathbb{G}}$ is a distinguished variety in $\tilde{\mathbb{G}}$ if and only if there exists a distinguished variety V in \mathbb{D}^2 such that $\tilde{W} = \tilde{\pi}(V)$.

Keywords: Distinguished variety, Inner toral polynomials, Symmetrisation map, Symmetrised bidisc