

Abstract No. 05

Mathematics

**COMPUTING h - FUNCTION OF THE COMPLEMENT OF A SLIT VIA
PARALLEL-SLIT MAPPING AND VALIDATION VIA DIFFERENT CONFORMAL
MAPPING**

M. Arunmaran^{*} and P. Piriyalucksan

Department of Mathematics and Statistics, University of Jaffna, Jaffna, Sri Lanka

^{}marunmaran03@gmail.com*

In potential theory, h -function is a concept intimately connected to Brownian motion and serves as a method for depicting the geometry of a domain. Consider a particle released from a fixed location z_0 in a region Ω and permitted to move randomly throughout Ω , exhibiting Brownian motion, until it first encounters the boundary. The distance from the starting point z_0 at the time of impact is recorded. Also, the process with millions of Brownian particles that were discharged from the same point z_0 is repeated. This information is formally expressed as a function known as the h -function of the given region Ω with respect to the specified basepoint z_0 . The h -function is an increasing function and takes only the values in the interval $[0,1]$. This study forms the h -function of a simply connected region Ω formed by deleting a slit from the complex plane when the basepoint z_0 is fixed along the line of the slit, by two different methods. The first method starts with *parallel-slit mapping*, which is in terms of the prime function. This map transforms the interior of the unit disc D_ζ to the region Ω . A *Cayley-type map* is used to transform the region D_ζ to the lower half-plane and the boundary of the disc to the real line. A function $W(\zeta)$ is formulated whose imaginary part is harmonic, and the $\text{Im}[W(\zeta)]$ evaluated at ζ_0 produces the h -function of the region Ω , where ζ_0 is the preimage of the basepoint z_0 . The second method is used to cross-check the formed h -function. In this method, a sequence of conformal maps whose composition transforms the given region Ω to the half-plane is used instead of the prime function. Subsequently, the intersection between the boundary of the region Ω and the closed ball of radius r centered at z_0 and its image in the half-plane is traced. Both methods produce the same h -function.

Keywords: Cayley-type map, h -function, Parallel-slit mapping