OPTIMAL TUNING OF ANTI-WINDUP COMPENSATORS (AWCs)

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ABSTRACT

The anti-windup compensators (AWCs) are techniques that are introduced to avoid the windup phenomenon in the poorly behaved modes of a controller when the actuator gets saturated. Unfortunately there are no proper guide lines to design or to tune the parameters of these AWCs, and especially when the order of the controller is high the manual tuning becomes very difficult and time consuming. In this research paper, the optimal tuning of the AWCs using numerical optimization methods is proposed and their effectiveness is investigated

INTRODUCTION

The windup phenomenon in controllers occurs when one or more of the controller modes keep on increasing (or decreasing) the controller output when the actuator get saturated. For example, in a standard Proportional plus Integral plus Derivative (PID) controller the integrator mode would keep on integrating the error even when the actuator is saturated and results in overshoots and oscillations in the output performance. The techniques that are used to reduce the effect of windup phenomenon are called anti windup compensators (AWCs).

The well known AWC methods are the conditioning technique (CT) presented in Hanus (1980) and the integrator resetting or dead-beat AWC (DAWC) presented in Vandenbussche (1975). However, as shown in Ronnback, Walgama and Sternby (1992) these techniques fail to give acceptable performance for some control systems. On the other hand the generalized anti-windup compensator (GAWC) proposed in Astrom and Wittenmark (1984) can be used with any general controller to prevent windup in all the modes of the controller. However, using GAWC means, for a general controller of order n it is necessary to tune n number of parameters. Unfortunately there are no proper guidelines to date to tune these n parameters. Hence, the main motivation for this research work is to investigate the possibility of using numerical optimization techniques to find the n parameters and thus to carry out optimal tuning of the AWCs.

It is shown in Ronnback, Walgama and Sternby (1992) that the GAWC is unable to give acceptable results when applied to some control systems. However, this conclusion was made by investigating only a restricted domain of the allowable poles of the GAWC. In order to improve the performance of the GAWC, they propose a further extension to the GAWC (called EGAWC) by introducing an additional polynomial. Though this modification has increased the number of parameters that has to be tuned, it was possible to tune this EGAWC to give acceptable performance even when the EGAWC poles are in a restricted domain.

The authors have demonstrated in the research work that a GAWC can be tuned to give acceptable performance for a class of control systems if its poles are not restricted and a numerical optimization method is used to search the complete allowable domain bounded by the unit circle. The limitations of the GAWC when applied to some control systems and the suitability of using numerical optimization to tune the EGAWC are also investigated.

THE CONTROLLER AND THE ANTI WINDUP COMPENSATOR

A general linear controller

A controller can be implemented either as an analogue or a digital controller. For the purpose of presentation, the digital controller is considered in this paper without lack of generality. A general 2-degree of freedom discrete (or digital) linear controller in the polynomial form is given by

$$R(q)u_k = T(q)w_k - S(q)y_k$$

Where w_k, u_k and y_k are the setpoint, control signal and the process output respectively and R, T and S are polynomials of the forward shift operator q, with appropriate orders. The different modes of $R(q)$ including the integrator and/or oscillatory modes can cause windup when control signal is saturated, and it is necessary then to introduce AWC measures to improve the performance.

Extended generalized AWC

It is shown in Walgama, Ronnback and Sternby (1992) that most of the different AWCs proposed in the literature can be expressed as special cases of the Extended generalized AWC (EGAWC) proposed in Ronnback, Walgama and Sternby (1992). Hence, only the EGAWC is presented here. For the general linear controller presented above, the EGAWC is given by

$$F(q)u_k = [\,F(q) - P(q)R(q)\,]v_k + P(q)[\,T(q)w_k - S(q)y_k\,]$$

where u_k and v_k are the controller output and the saturator output respectively, and the polynomials $F(q)$ and $P(q)$ are monic and their orders must be chosen to satisfy the condition $deg[F] = deg[PR]$. The polynomials F and P, which define the AWC, have to be chosen in a suitable way to give acceptable output performance. If the order of the controller is n, and if it is decided to use a m^{th} order P polynomial then the order of the F polynomial is $m+n$ and hence a total of $n+2m$ parameters have to be tuned.

When the polynomial $P(q)=1$ then we get the GAWC proposed in Astrom and Wittenmark (1984), and there are still n number of parameters that have to be tuned. Instead of tuning, a direct choice is to use the special cases of GAWC, namely the conditioning technique (CT) by Hanus (1980) and dead beat AWC (DAWC) by Vandenbussche(1975) . If $F=T(q)/t_0$, where t_0 is the coefficient of the highest order term in $T(q)$, then we get the CT, and if $F = q^n$ then we get the DAWC. However, these two methods would not give good results always and this is demonstrated in example 1.

Example 1

This example is taken from Walgama, Ronnback and Sternby (1992). The process is a discrete time double integrator system described by

$$(q - 1)^2 y_k = 0.5(1 + q)u_k$$

and the polynomial controller

$$u_k = \frac{0.36(q - 0.8)^2 w_k - (0.7876q^2 - 1.39q + 0.617)y_k}{(q - 1)(q + 0.206)}$$

is chosen to give a performance as shown in Fig. 1, when the controller output is not constrained. However, when the output of the controller is constrained to the interval *[-0.1 0.1]*, the controller gets windup and the system gets unstable. Hence we have to use an AWC. The performance of the system when CT and the DAWC is used is shown in Fig. 1. When the AWCs are used the system becomes stable, but the CT gives poorly damped oscillatory behaviour while the DAWC resulted in a sluggish response. This clearly demonstrates that these two choices of EGAWC, though straightforward are not always the best, and hence it is necessary to find techniques to tune the EGAWC.

Optimal tuning of AWCS

Since there are no proper guidelines to tune GAWC and EGAWC, in this research the authors resorted to use numerical optimization of an appropriate cost function to compute the optimal AWC parameters. The choice of cost function plays an important role, not only in the performance of the final control system, but also in the convergence properties of the optimization. Here, two of the cost functions that have been considered in the present investigations are presented.

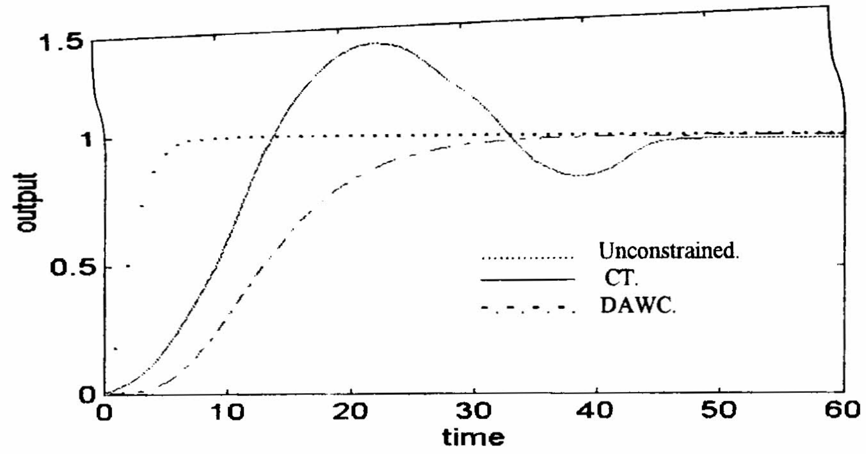

Fig.1. Performance with CT and DAWC for example 1.

Cost functions for optimization

$$\text{Sum of tracking error squared} = J_a = \sum_{i=0}^{N} (w_i - y_i)^2$$

Since taking the error square when the error is very small makes the contribution by these terms to the cost function to be insignificant, and often the optimal solutions with such cost functions result in systems that show some overshoots. This could be reduced by taking the

$$\text{Sum of absolute tracking error} = J_b = \sum_{i=0}^{N} |w_i - y_i|$$

In addition to the above two cost functions, optimization with cost functions based on weighted tracking errors and weighted nominal performance errors have also been considered. Cost functions have also been modified to include explicit overshoot terms.

Optimization technique

Considering the fact that these cost functions do not have analytical derivatives and also they may not be continuous, it was decided to use the optimization method based on simplex search method proposed in Nelder and Mead (1965).

Although this method may be less efficient than the quasi-Newton method, it may be robust for highly discontinuous systems. What is important to note here is that in the numerical optimization the sequence $\{y_i\}$ is obtained for a given set of parameters by explicitly simulating the system up to $i = N$. This in fact imposes heavy computation on the evaluation of the cost function.

RESULTS AND DISCUSSION

For the process and the controller given in example 1 earlier, a GAWC and an EGAWC are obtained by using the two cost functions and the numerical optimization method mentioned in the previous section.

The performance of the optimal GAWCs for the two cost functions is shown in Fig. 2. It is clear that the J_b which is based on the absolute error, results in an AWC that performed better than the AWC obtained when the error square criterion J_a is used. The AWC based on J_a gives rise to overshoots in the performance due to the inherent nature of J_a in giving very low weight to small errors when an overshoot occurs. For all the results presented from here onwards in this paper, only the cost function J_b is optimized to tune the AWCs.

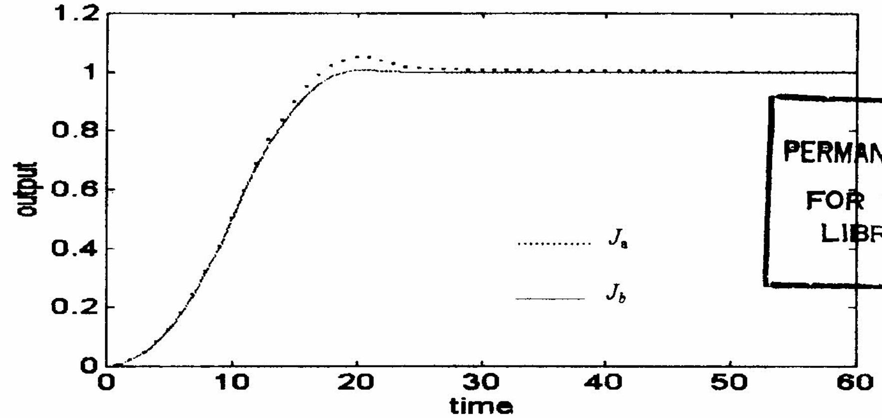

Fig.2. Optimal GAWC performance using J_a and J_b for example 1.

The numerical optimization gave an optimal GAWC that has the roots of the polynomial F at $q=0.7733$ and $q=0.6308$, which are real and distinct. For the optimal EGAWC, the polynomials P and L were chosen to be of order 2, and the other two roots of the F polynomial were chosen to be at $q=0.8$ as suggested in Ronnback, Walgama and Sternby (1992). The polynomials P and L were tuned using the numerical optimization method. The optimal P polynomial has the roots at $q=0.58+0.19j$ and $q=0.58-0.19j$, while the optimal L polynomial has the roots at $q=0.489+0.575j$ and $q=0.489-0.575j$, which are in fact complex. However, the performance of both GAWC and the EGAWC were almost the same. Although the EGAWC had more number of parameters to be computed than the GAWC, it was observed that the convergence of the optimization algorithm for EGAWC was much faster than for the GAWC.

Fig. 3 compares the performance of DAWC, and the optimal GAWC (or EGAWC). This also shows the performance of the EGAWC as reported in Walgama, Ronnback and Sternby (1992). The AWC in this case is obtained by hand tuning the polynomials L and P, which needs a considerable effort from the user. The hand tuned L polynomial has both its roots at $q=0.4$ and the P polynomial has both its roots at $q=0.7$. The hand tuning of the EGAWC was carried out by confining the roots to be real and coinciding. The hand tuning of the GAWC was quite difficult, because it was difficult to manipulate two parameters to get the best performance. On the other hand, the numerical optimization with appropriate cost function provides a convenient and efficient approach to tune both GAWC and EGAWC.

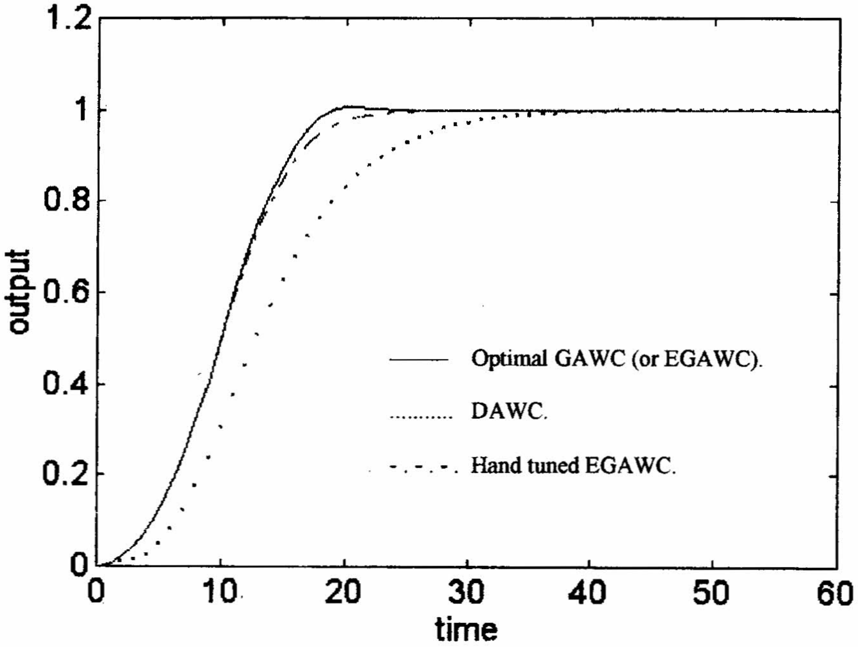

Fig.3. Optimal GAWC (or EGAWC) performance and handtuned EGAWC for example 1.

Since for the Example 1, both GAWC and EGAWC resulted in equal performance, to demonstrate the superiority of the EGAWC over GAWC, another example is considered next.

Example 2

This example is taken from Ronnback, Walgama and Sternby (1992). The process and the saturation limits to the input of the process are same as in example 1, but the controller is now given by

$$u_k = \frac{0.36(q - 0.97)^2 w_k - (0.4216q^2 - 0.821q + 0.399)y_k}{(q - 1)(q + 0.0492)}$$

The optimal GAWC has a *F* polynomial with roots at *q=0.987* and *q=0.895*, while the optimal EGAWC has a *L* polynomial with roots at *q=0.72+0.4j* and *q=0.72-0.4j*, and *P* polynomial with roots at *q=0.42* and *q=0.31*. The performance of both optimal GAWC and optimal EGAWC are shown in Fig. 4. In this case it is clear that the optimal EGAWC has better performance than the optimal GAWC. When proposing the EGAWC in Ronnback, Walgama and Sternby (1992), it was argued that the GAWC does not have the sufficient degree of freedom to give reasonable performance, and hence an additional degree of freedom was introduced in EGAWC. This argument was based on the simulation results where it was shown that by changing the *F* polynomial in the GAWC we can not get reasonable performance as with the EGAWC. However, changing the polynomial *F* which is of order *2*, is not easy in the complete domain where it is applicable, and hence in the investigations in Ronnback, Walgama and Sternby (1992), it was confined to coinciding real roots. In the present research, the complete domain is used for the optimization and has shown that even then the GAWC can not be tuned as good as the EGAWC for some control systems.

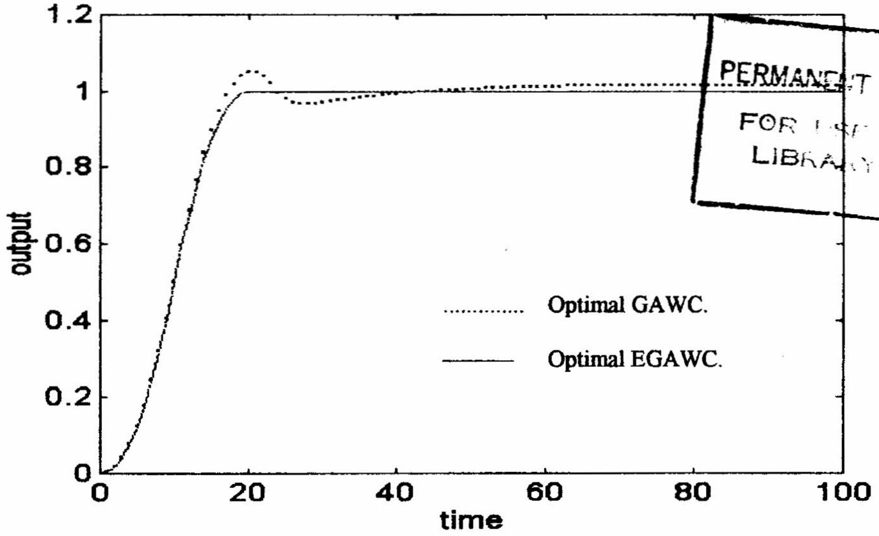

Fig.4. Optimal GAWC and optimal EGAWC performance for Example 2.

CONCLUSIONS

This research work has investigated the possibility of using numerical optimization methods to tune the AWC parameters in both GAWC and EGAWC. The authors have succeeded in tuning the GAWC to give a very good performance, which would have been very cumbersome and sometimes even impossible to tune manually, especially when the controller is of higher order. The main advantage in using the numerical optimization is that it provides the freedom to search in the complete applicable domain of the parameters of the AWC polynomials, whereas the hand tuning can be applied only to a very restricted sub domain.

The authors have also investigated a number of different cost functions that can be used to obtain the optimal solutions. This research has also confirmed the justification made in Ronnback, Walgama and Sternby (1992) to introduce the EGAWC, by demonstrating the limitations of the GAWC to achieve performance as good as that can be achieved by the EGAWC.

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