Abstract No. 53 ***Mathematics***

REFINEMENT OF REGULA FALSI METHOD FOR SOLVING NON-LINEAR EQUATIONS

H.H.S. Ziya*, W.A. Gunarathna and M.A.M. Mohamed

Department of Physical Science, Rajarata University of Sri Lanka, Mihintale, Sri Lanka
**safwahasan319@gmail.com*

Searching for roots of non-linear equations in the real domain is a common problem in science and engineering. Even though a few closed-form analytical solutions exist for algebraic equations, there is no such formula for transcendental equations, which brings numerical approximate methods to the frontline. The classical Regula Falsi method (M_1) is a root-finding numerical technique with linear convergence and requires checking the bracket condition at each iteration. Recently, a new root-finding technique (M_2) is proposed, that is superlinearly convergent with order $p = \sqrt{2}$, and it does not require checking the bracket condition. The iterations of this method are advanced by the formula : $x_{n+1} = \lambda N_{n-1} + (1-\lambda)N_n,\ 0 \leq \lambda \leq 1$ for $n = 1, 2, 3, \ldots$, where $N_m = x_m - f(x_m)/f'(x_m),\ m = 0, 1, 2, \ldots$, where $f'(x)$ is the derivative of the function defined in the non-linear equation $f(x) = 0$. In fact, due to the parameter λ, M_2 generates a family of root-finding techniques, all of which are superlinearly convergent. The convergence analysis of this method has also been established. However, this method fails in some cases, such as when 1) Nonlinear functions are not sufficiently smooth, 2) Nonlinear functions are not explicitly known, and 3) Derivatives have complex formulas. To alleviate those issues, each term N_{n-1} and N_n is replaced by the difference quotient D_{n-1} and D_n, respectively, to obtain the following method: (M_3) $x_{n+1} = \lambda D_{n-1} + (1-\lambda)D_n$, for $0 \leq \lambda \leq 1$ for $n = 1, 2, 3, \ldots$, where $D_m = x_m - [f(x_m)\big(f(x_m) - f(x_{m-1})\big)]/(x_m - x_{m-1})$ for all $m = 1, 2, 3, \ldots$. The accuracy and convergence efficiency of the proposed method M_3 are demonstrated by several numerical test examples. While M_3 achieves a better accuracy comparable to both M_1 and M_2, it converges to exact roots faster than both M_1 and M_2. For example, considering the initial approximations and $x_0 = -0.4$, $x_1 = 0.5$ for the exact root $r = 0$ of the equation $e^x \sin x + \ln(x^2 + 1) = 0$, M_1 requires 51 iterations to give an approximation of 0.000000000000009, M_2 achieves of an approximation 0.0000000000000000 within 16 iterations, while M_3 reaches an approximation of 0.0000000000014123 in 12 iterations.

Keywords: Non–linear equations, Order of Convergence, Regula Falsi Method, Roots