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JACOBI ELLIPTIC FUNCTIONS FROM THE PERSPECTIVE OF QUADRATIC DIFFERENTIALS

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Trigonometric functions are defined using the circle and are periodic with a single period along the real line. When the circle is replaced with an ellipse, a new class of functions can be introduced. By extending the independent variable to complex numbers, doubly periodic functions known as elliptic functions can be obtained. These have two distinct periods and are defined on the complex plane. Due to their doubly periodic nature, elliptic functions repeat their values over a fundamental parallelogram. In other words, their domain can be considered as a torus. Jacobi's elliptic functions $pq(u, m)$ are a generalisation related to ellipses. When the parameter m is 0 or 1, the functions reduce to non-elliptic forms. Although several types of elliptic functions have been comprehensively investigated, there has been a notable absence of studies on the quadratic differentials of elliptic functions. A quadratic differential refers to a meromorphic differential form of degree two defined on a Riemann surface, and they play a significant role in visualising geometric aspects of meromorphic functions. Geometrically, a quadratic differential induces a field of trajectories on its domain, whose behavior provides insight into the location and nature of the zeros and poles of the associated meromorphic function. This study examines the trajectory structure of Jacobi elliptic sn function within their fundamental rectangle and on the torus, using Python for visualisation. In this study, we discuss the quadratic differential form of autonomous systems and verify that the Phase Portrait of solutions to an autonomous system with two variables coincides with the trajectory structure of the corresponding quadratic differential. Finally, we represent the Jacobi sn function as an autonomous system with two variables and confirm that the trajectory structure of solutions to these autonomous systems aligns with the trajectory structure of the Jacobi sn function, along with its respective fundamental parallelogram.

Keywords: Autonomous Systems, Jacobi Elliptic Functions, Meromorphic Differential, Quadratic Differentials, Trajectory structure