

## Graph Labeling and Harmonies: Odd and Even Labeling of Star Graphs

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This research paper delves into the realm of graph theory, focusing on the concept of odd and even harmonies labeling within the framework of star graphs. The star graph family, characterized by a central vertex connected to peripheral vertices resembling a star's shape, serves as the primary subject of exploration. Harmonies labeling, an intriguing concept, involves assigning labels to graph vertices and edges following specific rules. A graph is considered harmonies if an injective function can be established between its vertex set and the integers modulo  $q$  (where  $q$  represents the number of edges), while ensuring distinct edge labels. This paper introduces the notions of odd and even harmonies labeling. In the odd harmonies scenario, each vertex is assigned a distinct even number from 0 to  $2q-1$ . Notably, the sum of labels on adjacent vertices results in a unique odd number between 1 and  $2q-1$ . Conversely, the even harmonies scenario assigns even labels to vertices within the same range, yielding a unique even sum for adjacent vertices. Extending Rosenfeld's 1979 work on odd harmonies labeling, this study explores both odd and even cases. The inherent properties of star graphs make them an ideal focus. As bipartite graphs, they naturally divide vertices into two sets, with connections exclusively between sets. Additionally, star graphs exhibit planar attributes, allowing representation on a flat surface without edge crossings. In summary, this research enriches graph theory by investigating odd and even harmonies labeling in the context of star graphs. By capitalizing on the unique properties of star graphs, this study advances our understanding of the intricate relationship between graph structures and harmonies labeling, encompassing both odd and even scenarios.

**Keywords:** Even harmonies labelling, Graph theory, Odd harmonies labelling, Star graph