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COMPARING MAXIMUM LIKELIHOOD AND BAYESIAN ESTIMATORS BASED ON WEIGHTED SQUARED ERROR AND ABSOLUTE ERROR LOSS FUNCTIONS FOR THE POISSON PARAMETER ESTIMATION

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The Poisson distribution is a standard model for count data, and several estimators are available for estimating the Poisson parameter λ. Among these, the Maximum Likelihood Estimator (MLE) is widely used. Alternatively, since the gamma distribution is defined on $(0, \infty)$, Bayesian estimators based on a gamma prior under various loss functions can also be employed. The gamma distribution, commonly used as a prior, is defined by a shape parameter a and a scale parameter b. However, comparative studies of such estimators with MLE are limited in the literature. This study evaluates the performance of the Bayesian estimators under different loss functions, including Squared Error Loss (SEL), Quadratic Loss (QL), Absolute Error Loss (AEL) and Weighted Squared Error Loss (WSEL), and compares them with the MLE through a comprehensive simulation study using the Mean Squared Error (MSE) criterion. To examine their performance in various small sample sizes n and some possible various parameter settings of a, b and λ simulation considered $n = 4, 8, 12, \ldots, 48$; $a = 0.5, 2.5, 4.5$; $b = 0.5, 2.5, 4.5$; and $\lambda = 0.5, 1.0, 1.5, \ldots, 10$ resulting in 2160 different scenarios. Under each scenario, 1,000 Poisson-distributed datasets were generated to provide a robust basis for assessing estimator performance. For the above given situations, the findings indicate that, when the sample size is large ($n \geq 50$) the MSEs of all estimators are almost zero across all settings. However, for smaller sample sizes ($n < 50$), the performances are different. Specially. MLE is more effective and consistently outperforms compared to the Bayesian estimators when $5 \leq \lambda \leq 10$ For smaller values of λ ($\lambda < 5$), Bayesian estimators showed superior performance, with their effectiveness influenced by the prior parameters' values a and b. For some specific settings of a and b, AEL and WSEL perform better than the QL and SEL. To validate the simulation findings, three real-world datasets were considered, covering above mentioned ranges of n and λ. The results demonstrated a strong alignment between the simulations and practical applications. Since the SEL, QL, AEL, and WSEL functions are not in closed form with respect to the parameters, it is difficult to mathematically define the range of parameters for which a particular estimator performs well. Further, this study method can be extended by incorporating other possible estimators for λ. Overall, the research highlights the strengths and applicability of both MLE and Bayesian estimators across various ranges of λ.

Keywords: Bayesian estimation, Gamma distribution, Loss functions, Maximum likelihood estimator, Poisson distribution