

**ASYMPTOTIC DYNAMICS OF A PERIODIC TOXIN-PRODUCING  
PHYTOPLANKTON MODEL**

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Harmful algal blooms (HABs) caused by Toxin-Producing Phytoplankton (TPP) have become increasingly common worldwide. Understanding the complex interactions between TPP and other organisms in the ecosystem is crucial. This study focuses on asymptotic dynamics of the plankton interaction between TPP and zooplankton, with an additional mortality factor accounting for zooplankton ingestion of TPP. One of the proposed models assumes constant toxin production by TPP, and the populations being studied are distributed uniformly throughout the space. This assumption results in an autonomous system of ordinary differential equations. To incorporate natural day and night, tidal, or seasonal cycles, the model is extended to a periodic system by including the toxin liberation rate  $\gamma$  as a periodic function of the form  $\gamma(t) = \gamma(1 + Aq(t))$ , where  $A$  represents the magnitude of the periodicity, and  $q(t)$  is a  $\tau$ -periodic function with  $\tau > 0$ . The study examines the existence of steady states and trivial periodic solutions and analyses the stability of both models. Moreover, using the concept of uniform persistence, we derive sufficient conditions for the coexistence of the periodic system based on the model parameters. The findings of this study demonstrate that increasing the avoidance of TPP by zooplankton enhances the coexistence of plankton populations in both the constant and periodic toxin production systems. The study also discusses Hopf bifurcations in the constant toxin-producing model, providing insights into the system's dynamic behaviour. Numerical simulations are performed to validate the analytical findings of proposed models and their implications.

**Keywords:** Harmful algal blooms, Periodic systems, Toxin-Producing Phytoplankton, Uniformly persistence, Zooplankton