

GENERALIZED CLIQUE COMPLEXES

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Networks are essential for modelling and analyzing complex systems in fields such as biology, social sciences, and finance, offering insights into the interactions and relationships within these systems. Despite their widespread use, classical graph-theoretic tools have inherent limitations, as they primarily focus on local properties, such as node degrees, paths, and direct connections between edges. This localized perspective is inadequate to capture global structural features such as complex connectivity patterns, holes, cycles, and higher-dimensional voids, which are crucial for understanding the overall network behaviour. Topological approaches in network analysis extend classical graph-theoretic methods by incorporating concepts from algebraic topology, enabling the study of higher-dimensional and global network features. Clique complex construction assigns a simplicial complex to a graph by taking complete graphs spanned by vertices as simplices. This approach enables the study of connectivity and other geometric features of the graph. This study generalizes the clique complex construction by using the notion of k -plex, an almost complete graph that spans n vertices. A k -plex is a subgraph on n vertices in which each vertex is at least of degree $n - k$. Also, this study proves that the assignment of a graph to its generalized k -clique complex is a functor from the category of graphs to the category of simplicial complexes. k -generalized clique complexes offer a more flexible representation of higher-dimensional interactions. Once the generalized k -clique complex is established, homology can be computed in the usual manner. The generalized k -clique complex construction ignores smaller holes and its higher dimensional counterparts, and depending on the choice of k , one has the choice of determining the size of the hole to be ignored. This approach will give a different perspective on network analysis and its applications.

Keywords: Clique Complex, k -plex, Persistent Homology, Simplicial Complex