

PERFORMANCE OF MAXIMUM LIKELIHOOD AND LINDLEY PRIOR BAYESIAN ESTIMATOR FOR THE POISSON DISTRIBUTION

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Accurate parameter estimation is essential for reliable statistical modelling. The Poisson distribution is commonly used to model count data, with the traditional method for estimating its parameter is the Maximum Likelihood Estimation (MLE) method. However, the effectiveness of the MLE method can diminish when data is limited and the mean is small, a situation often associated with a higher proportion of zero counts (zero-inflation). Such scenarios frequently arise in fields such as actuarial science, biomedical research, and environmental studies. In this context, one can resort to other types of estimators as an alternative to the MLE method. This study investigates and compares the performance of the MLE method and the Bayesian estimation method under a Lindley prior based on squared error loss and quadratic loss functions in various situations. The focus is on scenarios involving small sample sizes and low mean values. A comparative simulation study was conducted to compare these estimators using the mean squared error as the evaluation criterion. To formulate various situations, several Poisson rate parameters ($\lambda = 0.03, 0.05, 0.07, \dots, 9.99$), sample sizes ($n = 10, 20, 30, \dots, 180, 190, 200$), and Lindley prior shape parameters ($\theta = 0.25, 0.5, 0.75, \dots, 10$), and under each situation, 1000 Poisson-distributed samples were generated to improve the precision of the estimation. Then, in total, 399,200 situations were considered in this study. Results obtained from the given situations show that the Bayesian estimators based on squared error loss and quadratic loss functions outperform MLE for small sample sizes ($n < 100$) and very low mean values ($\lambda < 0.05$) and their performances are the same in all reviewed situations. Conversely, for larger sample sizes and higher mean values, MLE provides superior performance. To examine the consistency of the simulation study with real-world applications, two real datasets were considered covering the above ranges and found that the results are in line with the simulation findings. This study is limited to the situations and the Lindley prior considered above. It could be extended to cover other possible scenarios and priors. These findings highlight the advantage of the Bayesian estimation method in scenarios involving limited count data with a smaller mean.

Keywords: Bayesian estimation, Lindley prior, Loss functions, Maximum likelihood estimation, Poisson distribution