Abstract No: 72 ***ICT, Mathematics, and Statistics***

NEW THIRD-ORDER APPROXIMATION FOR FRACTIONAL DERIVATIVES

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Fractional derivatives (FDs) have seen diverse applications over the past few decades in many fields, including physics, biology, finance, and engineering. They are particularly useful for modelling memory and hereditary properties, which are essential for understanding materials and processes with memory effects that integer-order derivatives cannot capture. The non-local nature of FDs introduces computational challenges, including reduced accuracy, inefficiency, and instability, especially when solving fractional-order differential equations numerically. The Grünwald approximation (GA), which provides a first-order approximation of FDs, often results in unstable numerical solutions for fractional-order differential equations, like the space fractional diffusion equation. The shifted GA offers improved stability but retains only first-order accuracy. Similarly, Lubich approximations for FDs fail to provide stable solutions and revert to first-order accuracy when shifted. A second-order approximation was recently developed and applied to the space fractional diffusion equation, showing promising stability. Building on this, third-order and fourth-order approximations have been derived from the second-order method, albeit in quasi-compact form. This study proposed a new third-order accurate approximation derived from the second-order approximation, utilizing a convex combination of the second-order approximation with two different shifts, r_1, r_2. The proposed third-order approximation is applied to a one-dimensional steady-state problem with numerical test examples. The results indicate that the third-order approximation is significantly more accurate than the second-order approximation, achieving a convergence order of 3, consistent with the theoretical order. For instance, with a grid size $h = 1/2048$ and fractional order $\alpha = 1.5$, using $r_1 = -1$ and $r_2 = 1$, the maximum error of the proposed approximation is 4.20E-08, compared to 1.63E-05 for the second-order approximation. The proposed method has potential applications for solving super-diffusion problems in various fields, such as finance and physics.

Keywords: Fractional derivatives, Generating functions, Grünwald approximation, Space fractional diffusion equation