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# TOWARDS AN ALGEBRA OF HYPERMATRICES: A HYPERMATRIX REPRESENTATION FOR MULTILINEAR MAPS

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Matrix theory offers a complete and well-understood framework in which linear transformations are represented by matrices, with composition realised through matrix multiplication. However, no such canonical multiplication exists for hypermatrices that extends this structure to the multilinear setting, primarily due to the failure of multilinear maps to be closed under composition. To address this gap partially, the present work introduces a novel multiplication between two hypermatrices with a particular compatibility. The present work establishes a bijective correspondence (modulo choice of ordered bases for each vector space) between multilinear maps $\boldsymbol{f : V_1 \times V_2 \times \cdots \times V_n \rightarrow V_0}$ and hypermatrices $\boldsymbol{\mathcal{A}}$, by developing a multiplication operation between $\boldsymbol{\mathcal{A}}$ and a hypermatrix derived from $\boldsymbol{V_1 \times V_2 \times \ldots \times V_n}$. Since multilinear maps, in general are not composable, the multiplication introduced does not extend to a general multiplication of hypermatrices. This study provides both explicit computational procedures and theoretical proofs. Beyond partially extending matrix algebra to the multilinear case, this result offers new insights into the structural relationship between hypermatrices and multilinear operators. Moreover, the proposed multiplication has the potential of serving as a foundational operation for a coherent algebraic theory of hypermatrices, analogous in structure and purpose to classical matrix algebra.

**Keywords:** Canonical multiplication, Hypermatrices, Linear transformation, Multilinear algebra