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SIXTH-ORDER RUNGE-KUTTA DISCRETIZATION OF THE QUADRATIC RICCATI EQUATION

B.H.O. Jayasinghe*, W.A. Gunarathna and M.A.M. Mohamed

Department of Physical Sciences, Rajarata University of Sri Lanka, Mihintale, Sri Lanka
**jhimanthi@gmail.com*

The Quadratic Riccati Differential Equation (QRDE) is a nonlinear ordinary differential equation that has many significant applications in computational sciences including optimal control theory, biological sciences, stochastic processes and physics. The initial value problem for the QRDE, frequently appearing in applications, is given by $y'(t) = p(t) + q(t)y + r(t)y^2$, $y(t_0) = \alpha \in \mathbb{R}$, where $p(t)$,$q(t)$, and $r(t)(\neq 0)$ are continuous functions defined on the interval $t_0 \leq t \leq T$. The nonlinear term y^2 in the QRDE often presents challenges, including lack of closed-form solutions, complex solution forms, and singularity problems. As a remedy, computational methods for the QRDE have been developed in literature. However, existing numerical methods such as Euler and Taylor series often fail to meet the numerical accuracy demanded by most applications. For example, the forward and backward Euler methods have first-order accuracy. Taylor series methods need the computation of higher-order derivatives. The classical fourth order Runge-Kutta has been utilized by File and Aga to obtain numerical solutions with an accuracy of order four. Using the fifth order Runge-Kutta method, we have recently obtained fifth order accurate solutions for the QRDE. In this study, a sixth order variant of Runge-Kutta method is applied to discretize the QRDE with a given initial condition. The stability and convergence of this method were also analyzed. Several numerical tests were carried out to illustrate the accuracy of the proposed method. Here, maximum absolute errors with respect to exact solutions on uniform grids of the domain $[t_0, T]$ with given t_0 and T were calculated for each example. The numerical results confirm the accuracy of the derived sixth order method. For example, for a grid length $h = 0.0025$, the maximum absolute errors for the numerical tests are 9.3940e-04, 5.1002e-05, 5.3373e-08 and 7.3614e-06.

Keywords: Accuracy order, Initial-value problems, Quadratic Riccati equation, Runge-Kutta methods