

PRIME FACTOR COUNTING METRIC ON POSITIVE RATIONAL NUMBERS

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Up to the same topology, there are only three real absolute values on the rational numbers: the discrete metric, the Euclidean metric, and the p-adic metric. This work introduced a novel metric for positive rational numbers, henceforth referred to as the prime factor counting metric, which is based on their unique prime factorizations and is compatible with multiplication. While this metric has been successfully established for positive integers before, this study extends it by utilizing the greatest common divisor and least common multiple operations for positive rational numbers. The study showed that the extension retains the properties of a metric and provides a unit distance criterion based on prime factors. The criterion offers an intuitive method for calculating the distance between positive rational numbers through prime factor adjustments. The proposed metric induces the discrete topology on \mathbb{Q}^+ , which can be better described using a Hasse diagram and the associated graph metric, where each edge represents multiplication or division by a prime number.

Keywords: Hasse diagrams, Metric spaces, Prime factor counting metric, Real absolute values