

TWO TYPES OF BI-FUZZY OPEN SETS IN BI-FUZZY TOPOLOGICAL SPACES AND THEIR PROPERTIES

M. Arunmaran* and P. Piriyalucksan

Department of Mathematics and Statistics, University of Jaffna, Jaffna, Sri Lanka.

*marunmaran03@gmail.com

The idea of fuzzy topological spaces was initially proposed by Chang in 1968, inspired by Zadeh's discovery of fuzzy sets. In his work, the fundamental concepts of classical topology were examined to create a new theory of fuzzy topology integrating fuzzy sets. More recently, the notion of bi-fuzzy topological spaces has emerged. This study aimed to explore two novel types of open sets within bi-fuzzy topological spaces, namely, bi-fuzzy (m, n) - α_n -open sets and bi-fuzzy (m, n) - β_n -open, and elucidate some properties associated with these open sets. For a non-empty set X , a fuzzy topology is a family τ of fuzzy subsets in X satisfying the following conditions: $0_X, 1_X \in \tau$; finite intersection of members of τ is a member of τ ; and the arbitrary union of members of τ is a member of τ . The pair (X, τ) is called a fuzzy topological space. Also, the elements of τ are called fuzzy open sets. Moreover, we call the triple (X, τ_m, τ_n) as the bi-fuzzy topological space, where τ_m and τ_n are two fuzzy topologies defined in a non-empty set X . Let (X, τ_m, τ_n) be a bi-fuzzy topological space, and A be a fuzzy set. Then the set A is called bi-fuzzy open if $A \in \tau_m \cap \tau_n$; bi-fuzzy- (m, n) -preopen, if $A \leq \text{int}_m(\text{cl}_n(A))$; bi-fuzzy- (m, n) -semi-open, if $A \leq \text{cl}_m(\text{int}_n(A))$; bi-fuzzy- (m, n) - α_n -open, if $A \leq \text{int}_n(\text{cl}_m(\text{int}_n(A)))$; bi-fuzzy- (m, n) - β_n -open, if $A \leq \text{cl}_n(\text{int}_m(\text{cl}_n(A)))$. In this study, firstly it was demonstrated that a bi-fuzzy- (m, n) - α_n -open set is distinct from a bi-fuzzy- (m, n) - α_m -open set. Subsequently, we established that the union of two bi-fuzzy- (m, n) - α_n -open sets is a bi-fuzzy- (m, n) - α_n -open set. However, the intersection of two bi-fuzzy- (m, n) - α_n -open sets does not yield a bi-fuzzy- (m, n) - α_n -open set. Furthermore, we illustrated that every bi-fuzzy open set is a bi-fuzzy- (m, n) - α_n -open. Also, every bi-fuzzy- (m, n) - α_n -open set is bi-fuzzy- (m, n) -semi-open. However, the converse of these results is not true. Next, we showed that the union of two bi-fuzzy- (m, n) - β_n -open sets is a bi-fuzzy- (m, n) - β_n -open set. However, the intersection of two bi-fuzzy- (m, n) - β_n -open sets need not be a bi-fuzzy- (m, n) - β_n -open set. It is established that every bi-fuzzy open set is a bi-fuzzy- (m, n) - β_n -open. Finally, we showed that every bi-fuzzy- (m, n) -preopen set is bi-fuzzy- (m, n) - β_n -open. In this study, within a bi-fuzzy topological framework, two new types of open sets were introduced, namely bi-fuzzy- (m, n) - α_n open set and bi-fuzzy- (m, n) - β_n -open sets. Also, the relationships between the union and intersection of these fuzzy sets were explained. Moreover, the connections between these two sets and the following categories were explored: bi-fuzzy- (m, n) -preopen sets, bi-fuzzy- (m, n) -semi-open sets and bi-fuzzy open sets.

Keywords: Bi-fuzzy (m, n) - α_n -open sets, Bi-fuzzy (m, n) - β_n -open sets, Bi-fuzzy (m, n) -preopen sets, Bi-fuzzy (m, n) -semi-open sets, Bi-fuzzy topological spaces