

HOPF ALGEBRA OF LABELED SIMPLE GRAPHS, PERMUTATIONS AND POSETS WITH HOMOMORPHISMS AMONG THOSE

S.M.R.N. Mudalige* and K.M.N.M. Chathuranga

Department of Mathematics, University of Peradeniya, Peradeniya, Sri Lanka.

**rashinim1999@gmail.com*

A Hopf algebra is a compatible bialgebra equipped with an additional map called the antipode map. This study explored the Hopf algebra structures on three fundamental combinatorial objects: Labeled simple graphs (\mathbb{H}), Permutations (\mathbb{S}), and Posets (\mathbb{P}). The main objective was to study three particular Hopf algebra homomorphisms $f: \mathbb{S} \rightarrow \mathbb{P}$, $g: \mathbb{S} \rightarrow \mathbb{H}$, and $h: \mathbb{P} \rightarrow \mathbb{H}$. It is shown that only subsets of labeled simple graphs (those with more than one vertex and at least one edge, as well as the empty graph) and posets arise under these homomorphisms. The composition homomorphism of f and h denoted by $hf: \mathbb{S} \rightarrow \mathbb{H}$, behave differently from g . The set of all permutations in ascending order, along with the empty permutation, forms the kernel of hf . Similarly, the totally ordered posets and the empty poset constitute the kernel of h . This study contributes to the field by providing insights into how homomorphisms preserve or distort the three combinatorial Hopf algebra structures. The findings not only enhance the understanding of Hopf algebras in combinatorics, but also lay the groundwork for further research in applications in algebraic combinatorics. Future work will focus on developing explicit formulas for the antipodes deepening the understanding of combinatorial structures within the combinatorial Hopf algebraic framework.

Keywords: Combinatorial Hopf algebra, Hopf algebra homomorphisms, Labeled simple graphs, Permutations, Posets