RESCON 2025 CMS 14

MATRIX REPRESENTATION OF DISTINGUISHED VARIETIES ON SYMMETRISED BIDISC

S. Perera* and U.D. Wijesooriya

Department of Mathematics, University of Peradeniya, Peradeniya, Sri Lanka.
*S19475@sci.pdn.ac.lk

The symmetrisation map $\Pi(z,w) \to (z+w, zw)$ is the map given by $\Pi(z,w) \to (z+w, zw)$ and the symmetrised bidisc, $\mathbb{G}$ is given by $\Pi(\mathbb{D}^2)$, where $\mathbb{D}$ is the open unit disc in $\mathbb{C}$. A non empty set $\mathrm{W} \subseteq \mathbb{C}^2$ is called a Distinguished Variety on $\mathbb{G}$ if there exists a polynomial $q(s,p) \in \mathbb{C}[s,p]$ such that $W = \{(s,p) \in \mathbb{G}: q(s,p) = 0\}$ and $Z(q) \subseteq \mathbb{G} \cup b\mathbb{T} \cup \Omega$ where $b\Gamma$ is the image of $\mathbb{T}$ under Π and Ω is the image of $\mathbb{E}$ under Π. It was proven that, any Distinguished Variety on $\mathbb{G}$ can be represented via a square matrix with numerical radius $(\omega(A))$ less than 1. Conversely, given a Distinguished Variety on $\mathbb{G}$ there exists a square matrix A with $\omega(A) \leq 1$ such that $W = \{(s,p) \in \mathbb{G}: \det(A + pA^* - sI) = 0\}$. In this work, we prove that if A_k is a square matrix representing the Distinguished Variety W_k on $\mathbb{G}$ for $k = 1, \ldots., n$ then, $\bigoplus_{k=1}^{n} A_k$ represents the Distinguished Variety $W = \bigcup_{k=1}^{n} W_k$ on $\mathbb{G}$. This result would allow us to generate examples for matrices representing Distinguished Varieties on $\mathbb{G}$, specially for reducible Distinguished Varieties, by taking direct sums of the matrices representing its irreducible components.

Keywords: Distinguished variety, Numerical radius, Symmetrised bidisc