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FIFTH-ORDER RUNGE-KUTTA METHOD FOR SOLVING QUADRATIC RICCATI EQUATION

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The quadratic Riccati Differential Equation (RDE), a non-linear ordinary differential equation, has found numerous applications in many fields including biology, finance, and engineering. Solving the RDE analytically is challenging due to several reasons such as the lack of closed-form solutions, complex solution forms, and singularity issues. Numerical methods are often preferred for solving RDE. The initial-value problem for RDE, denoted by IVPRDE, with the initial condition $y(t_0) = \alpha \in \mathbb{R}$, is given by $y'(t) = p(t) + q(t)y + r(t)y^2 = f(t, y)$, where $p(t), q(t)$, and $r(t)(\neq 0)$ are continuous functions defined on the interval $[t_0, T]$. Recently, the classical fourth-order Runge-Kutta (RK4) method has been applied to obtain numerical solutions for IVPRDE, establishing its stability and convergence. In literature, higher-order (> 4) Runge-Kutta methods have not yet been applied to solve the IVPRDE. This study aimed to apply a fifth-order Runge-Kutta method, specifically Butcher's fifth-order Runge-Kutta (BRK5) approximation, to numerically solve the IVPRDE and obtain its numerical solutions. The stability and convergence of the proposed method were established. Using BRK5, maximum errors on a discretized domain of $[t_0, T]$ with different uniform grid sizes (h) were computed and compared with those of RK4 method for four numerical test examples. Numerical results demonstrate that BRK5 has a superior accuracy over RK4. For example, when $h = 0.01$, the maximum errors due to BRK5 for the tests are 3.1086e-15, 1.8797e-12, 8.8818e-16, and 7.4052e-14, whereas the corresponding errors computed by RK4 are 2.6645e-15, 1.4105e-09, 6.5340e-10, and 6.7954e-11. These results verify the effectiveness of BRK5 over RK4 for solving IVPRDE.

Keywords: Higher order Runge-Kutta methods, Initial value problems, Numerical solution, Riccati differential equation