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CHARACTERIZATION OF INNER TORAL POLYNOMIALS VIA FINITE BLASCHKE PRODUCTS

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A finite Blaschke product is a product of finitely many automorphisms on the unit disc. These complex-valued functions are bounded and analytic on the unit disc. An inner toral polynomial is a polynomial in $\mathbb{C}[z,w]$ such that its zero set is contained in $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where $\mathbb{D}$ is the open unit disc, $\mathbb{T}$ is the unit circle, and $\mathbb{E}$ is the exterior of the closed unit disc in $\mathbb{C}$. Finite Blaschke products generate inner toral polynomials in the following way; given a finite Blaschke product $B(z)$, the numerator of $w^m - B(z)$ is an inner toral polynomial. Previous work has shown that every inner toral polynomial of the form $q(z)w^m - \alpha r(z)$, where $\alpha \in \mathbb{C} \setminus \{0\}$, $m \in \mathbb{N}$ and $r(z)$, and $q(z)$ are polynomials in z with deg $(q(z)) \geq$ deg $(r(z))$, is also generated by a finite Blaschke product. This study generalizes previous results by considering inner toral polynomials generated by two finite Blaschke products. It is proved that for given two finite Blaschke products $B_1(z)$ and $B_2(w)$, the numerator of $B_1(z) - B_2(w)$ is an inner toral polynomial. Furthermore, under certain limitations, a partial converse is also proved. If $p(z,w) = \alpha r_1(z)q_2(w) - q_1(z)r_2(w)$, where $\alpha \in \mathbb{C} \setminus \{0\}$, and $r_1(z)$, $q_1(z)$, are polynomials in z, and $r_2(w)$, $q_2(w)$ are polynomials in w, with degree one is inner toral, then $p(z,w)$ can be written as the numerator of the difference of two finite Blaschke products. These findings further deepen the understanding of the relationship between finite Blaschke products and inner toral polynomials, providing new insights into the structure of distinguished varieties and their generation in multivariable complex analysis.

Keywords: Distinguished varieties, Finite Blaschke products, Inner toral polynomials