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A Structural Comparison of Mathematical Models for Educational Planning.

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The mathematical models approach to educational planning is of recent origin. At most a decade has elapsed since its intervention. Nevertheless, the variety, diversity and complexity of present educational problems have resulted in the construction of a wide spectrum of mathematical models for educational planning. In this paper, using a classification framework, it is attempted to conduct a guided tour over a selected set of educational planning models representing a fair cross-section of the present state of the art and to carry out a structural comparison within them. The major analytical weaknesses in the educational planning models that have been brought to light by the structural analysis are subsequently discussed with a view to pointing out the possible future development of the models.

A Classification Framework

For a meaningful structural analysis of the mathematical models a classification framework is a prerequisite. There are several ways in which the large number of models can be classified. Although there have been five previous surveys of educational planning models,[1] only in the works of Correa (1967), Khan (1970) and Weathersby and Weinstein (1970), have classification procedures been utilised. The last two are possibly the soundest. Khan's survey, however, did not as a matter of deliberate choice include a number of model-types. In the framework created by Weathersby and Weinstein, though it is very detailed, some of the important characteristics of models covering both educational system and economy have been left out, since their survey was restricted to models for university planning. These considerations thus indicate the possibility of improving the existing classification frameworks.

After considering the past classification procedures the following framework has been adopted here. This is based on four major aspects of the model: (i) its function or purpose; (ii) the mathematical technique or method used in it; (iii) its subject matter; and (iv) its operational status. Each of these major categories is subdivided into several specific items as described below.

Function: A mathematical model aims to serve several different purposes. Some models are designed to 'derive' measures of system cost or activity; for example, cost per student, cost per degree and number of teaching hours per staff member. Alternatively, other models have the capacity to project or 'forecast' enrolments, staff required, or institutional expenditures for each year

1. These refer to: Correa (1967), Fox and Sengupta (1968), Thonstad (1969), Khan (1970) and Weathersby & Weinstein (1970).

in the planning horizon. Another aim of models may be to 'allocate' given resources among alternative uses. These aims are not mutually exclusive and it is conceptually possible to construct mathematical models to perform more than one of these functions.

Method: Mathematical models can be also classified according to the type of analytical techniques used. Here, four main techniques of analysis are distinguished: 'deterministic,' 'statistical,' 'optimization' and 'simulation.' Models based on input-output or accounting equations are categorized as 'deterministic,' while those based on either statistical techniques or involving stochastic variables are termed 'statistical.' Optimization models make use of the application of mathematical programming. Basically, simulation models attempt to associate cause with effect, action with reaction, policy with result, through the use of mathematical formulae. The term is often associated with numerical analysis in conjunction with electronic computers. It is also possible to construct models by incorporating more than one type of analytic technique.

Subject: The spectrum of subjects considered in mathematical models ranges from global considerations of the entire educational system seen in relation to the economy and labour market, to very specific partial studies which focus on a particular section of the educational system. It seems appropriate to classify the subject-matter of models under three broad headings. Models which encompass the educational system and the economy can be called 'comprehensive externally-linked' models, preserving 'comprehensive' models to identify those which deal with the whole or major part of the education system.

'Partial' models are the third broad category under which the rest of the models can be grouped. These models analyse one or several of the subjects within a given educational institute, or within a section of the educational system. The subjects may include 'student flow,' 'student achievement,' 'demand and supply of staff,' 'curriculum,' 'library facilities,' 'physical space' and 'financial factors.'

Status: The extent to which a given model is currently operational is also an important factor to consider. It seems reasonable enough to point out here whether a particular model is limited to 'research' effort or is 'currently operational.'

Structural Analysis of Models

On the basis of the above classification framework the accompanying Table presents in summary form the salient features of each of the forty models we have selected from the literature. These models represent a fair cross-section of the present state of affairs. This table also reflects the areas of concentration of present model building exercises as well as the areas where future attempts should be focussed.

For this analysis, the three broad headings upon which the subject-matter classification is based can be employed.

Comprehensive Externally-linked Models

There are only a few models belonging to this category. The Adelman model which referred to Argentina and was based on linear programming techniques is a cross between two basic approaches to educational planning—the manpower-forecasting approach and the cost-benefit analysis approach. As in the manpower-forecasting approach, the model generates its demands upon the educational system from the production side of the economy and uses fixed labour-output coefficients for each of three manpower categories; primary school graduates, secondary school graduates, and university graduates. Just as in the cost-benefit analysis approach, "the extent of resources to be devoted to education is determined in the model by comparing the marginal social benefits of each type of education with its marginal social cost." It assumes that all manpower formation is domestic. The optimization is also subject to a set of socio-economic constraints which stipulate that enrolments in each type of school be nondecreasing through time. The labour-availability-labour-use constraints provide the major link between the educational and the non-educational portions of the model. Even though the model is disaggregated into six educational activities and only nine productive sectors, it still includes 68 equations for each time period.

The Arnold-McNamara model stems from the works of Arnold and McNamara (1971) and McNamara (1971). The model, based on the systems approach, was initially developed in Pennsylvania and applied to its counties. It is designed to permit better utilization of available socio-economic data to improve decision making and programme planning in vocational and technical education within the broad sphere of socio-economic planning. In this model, the state vocational educational system has been viewed as a set of input-output or production relationships. By considering existing vocational educational programmes, the supply and demand of trained persons, the existing socio-economic conditions and trends, and available funds and resources, the Arnold-McNamara model makes use of linear programming techniques to calculate optimal solutions for specific decision making parameters. It is particularly interesting to observe how the authors overcome the common problem underlying the fitting of educational preparation to occupational requirements in the work force by using the commonly defined occupational language used by the U.S. Department of Labour and the U.S. Department of Health, Education and Welfare. The possibility of incorporating some of the stochastic nature of the parameters into the model in future has been pointed out by McNamara.

The Bernard model, proposed for France, is set out in the form of a sequential linear programme embracing the entire national economy, education being treated as one of its 'sectors' or 'activities.' Its purpose is to determine the optimum allocation of resources, mainly physical, between education and the 'commercial' economic activities represented by the sectors of an inter-industry input-output table. In this system, the educational sector is regarded as a producer of the knowledge needed for the training of skilled workers employed in all sectors. This optimized allocation is obtained by maximising, subject to constraints, a social preference function; this function is made up of numerical indices of the living standard of the population throughout the year considered, and of the production potential at the end of the period.

The Correa-Tinbergen-Bos model is a result of the first step taken towards constructing a formal model for educational planning integrated with

Classification of Mathematical Models for Educational Planning.

Nomenclature	Function			Method				Subject									Status	
										Partial								
	Derivation	Forecasting	Allocation	Deterministic	Statistical	Optimization	Simulation	Comprehensive externally-linked	Comprehensive	Student flow	Student achievement	Staff demand supply	Curriculum	Library facilities	Physical space	Financial	Research	Currently operational
Adelman (1966)			x			x		x									x	
Armitage *et., al.*, (1969)		x			x				x								x	
Arnold-McNamara			x			x		x										x
Asian: UNESCO (1966)		x	x	x					x									x
Baisuck & Wallace (1970)		x					x			x							x	
Benard (1967)			x			x		x									x	
Bowles (1965)			x			x			x								x	
Bowman (1969)		x	x		x		x									x	x	
Bruno (1970)	x						x					x					x	
CAMPUS: Judy & Levine (1965)	x	x			x		x			x		x		x	x	x		x
Clough (1971)	x	x			x	x					x						x	
Coleman-Mood-Mayeske	x				x				x									x
Correa (1965)	x					x							x				x	
Correa-Tinbergen-Bos			x	x				x										x
DYNAMOD II: Zabrowski (1969)		x			x		x		x								x	
Ewald & Kiker (1970)	x			x												x	x	
GAME: LeVasseur (1969)		x	x	x			x	x									x	
Gani (1963)		x		x						x							x	
Graves & Thomas (1971)			x			x									x			x
Harden & Tcheng (1971)		x			x	x				x								x
Hoenack (1969)			x		x	x										x	x	
JCL 3W: Judy (1969)	x	x			x		x			x		x	x		x			x
Kleindorfer & Roy (1969)		x	x	x			x		x									x
MSU: Koenig & Keeney (1969)	x	x	x		x		x			x		x		x	x	x	x	
Marshall & Oliver (1970)		x			x		x			x	x						x	
Marshall, Oliver & Suslow (1970)		x			x		x			x							x	
McReynolds (1971)			x		x		x			x	x						x	
Menges & Elstermann (1971)	x	x	x		x	x				x		x			x		x	
Morton (1969)		x		x								x						x
Murteira (1970)		x		x						x							x	
O'Brien (1969)			x		x	x					x	x			x	x	x	
Oliver (1968)		x			x					x							x	
Perl (1971)	x		x		x						x					x	x	
Pollard (1970)	x				x		x				x						x	
Reisman & Taft (1969)		x			x		x			x		x					x	
SD Two: Szekely *et., al.*, (1968)	x		x		x		x		x								x	
Stone (1972)	x	x		x					x								x	
Swedish Central Bureau of Statistics (1967)		x		x					x									x
Thonstad (1969)	x	x		x					x								x	
Werdelin (1970)		x			x					x							x	

the economy. The model originally formulated by Correa and Tinbergen (1962) was later revised and extended by Tinbergen and Bos (1964). Using a simple model of input-output type, they attempted to relate needed secondary and higher education outputs directly to given rates of economic growth, without using the intermediate step of calculating the occupational requirements. Essentially, the number of persons required from each educational level was calculated from a series of linear difference equations which related the stock of persons completing a given level of education and the number of students in each level to the aggregated volume of production. Its purpose was to suggest what structure of the educational system was needed in order to 'let the economy grow at a certain rate,' and how that structure should change with changes in the growth rate. The model is distinctively of the manpower planning type. Thus, the Mediterranean Regional Project (MRP) born under the OECD provided an excellent opportunity to test the model, and subsequently it received much attention. Several authors have investigated the application of the model to the educational programmes of Spain, Turkey, and Greece—three of the six countries included the MRP.[2]

The Global Accounts for Manpower and Education (GAME) model was constructed by the OECD for use in their Training Seminar held in 1967. The model considers simultaneously the education, manpower and economic systems. By considering the flow of human individuals as well as goods and services between and within the three systems, an integrated set of accounting equations has been formulated. In it, 25 educational branches and grades, 8 educational qualifications, 20 occupations and 23 industries have been distinguished. At this level of disaggregation, the model consists of 352 equations and 387 variables which have to be solved for each year of the planning horizon. In designing the GAME model, a conscious effort has been made to arrive at a 'generalised' formulation so that it is not confined to any particular educational environment. Owing to the incorporation of the facilities for man-machine iterative procedures, the model can be used to trace the effects of proposed alternative national educational policies upon the manpower and economic systems. The simulation exercises, carried out by the participants at the OECD Training Seminar, have demonstrated that only by an explicit consideration of the interdependent relationships among education, manpower and the economy can consistency in planning be achieved. Thus a model of this kind offers considerable potential for the formulation of a consistent national policy and plans in these areas.

To sum up, the overall comparison of comprehensive externally-linked models reveal that during their construction either deterministic or optimization techniques of analysis have been commonly used rather than statistical techniques. The treatment of uncertainty within this category of models has been weak. However, if repeated simultation is an acceptable substitute for analysis of uncertainty, the GAME model will be worth consideration.

Comprehensive Models

So far, as with comprehensive externally-linked models, there has been a limited effort in developing models extending over the entire educational system or a major part of it. One outstanding common feature of the comprehensive

2. See papers by L. J. Emmerij, J. Blum and G. Williams in OECD (1965).

models is that they are of Markov-type. This is observable in every model except in Asian, Bowles, Kleindorfer-Roy and in the model from the Forecasting Institute of the Swedish Central Bureau of Statistics.

The attempts of the Higher Education Research Unit at the London School of Economics and Political Science to develop a detailed computable model of the British educational system is one of the prominent efforts at constructing comprehensive educational models. As reported in Armitage *et al* (1969), subsequent to the construction of a Markov-type forecasting model, the Unit has attempted to introduce 'bottlenecks' or capacity models as an enlargement of it. At this stage, the consequences of bottlenecks have been discussed at some length using simple selection and re-allocation rules, and their effect upon the mathematical equations studied. The Unit expects, with their ongoing research, to reach results compatible with elements of decision making.

The Asian model was constructed by UNESCO in co-operation with EC AFE on the request of the Ministers of Education of Asian Member States who met in Tokyo in 1962. It consists of a set of accounting equations incorporating school enrolments in each grade, teacher requirements, enrolments in teacher-training institutions, literacy and adult education, and costs. The equations involve a large number of component factors and pertinent variables. The model is sufficiently detailed to permit adaptation to particular situations and is not invalid if all of the elements noted are either not available or not relevant within the framework of educational planning as it may be practised in a particular country. The Kleindorfer-Roy model is an adaptation of the Asian model to suit Bangladesh by excluding the 'literacy and adult education' sector in the Asian model. Since the Kleindorfer-Roy model is computerized, the consequences of choosing alternative policy parameters can be examined.

The model from the Forecasting Institute of the Swedish Central Bureau of Statistics is similar in structure to the Asian model but concentrates only on student population at schools and at universities, regardless of teacher requirements and costs.

The Bowles model was designed for use in Northern Nigeria. Treating inputs into the educational sector from the rest of the economy as exogenously determined, Bowles set up as his objective function the maximization of the excess of total economic benefits over costs of education. The problem is one of resource allocation within education. Furthermore, he tested for choices among alternative educational policies such as the effect of reducing the present seven-year primary school course to five years. As in the Correa-Tinbergen-Bos model, he classified labour directly by educational attainments. The model also assumes that each category of educated labour is highly substitutable with both other types of labour and with capital. In this respect the model differs significantly from the Correa-Tinbergen-Bos model which assumes that the production functions in the economy are characterised by fixed input coefficients for labour classified by educational level. Bowles' work is especially interesting in that he demonstrated for the first time the usefulness of programming techniques in the modelling and planning of an educational system.

The major factors affecting the learning process appear to be very much confounded. Certainly, there is so much interaction between home and formal institutions in learning that one can hardly expect to separate their effects fully.

The Coleman-Mood-Mayeske model for student achievement is an effort to disentangle these major factors. It is a result of an analysis of the data gathered in the U. S. Office of Education's Educational Opportunities Survey in 1965. The model originates from Coleman, Mood *et al* (1966)—the Coleman Report—supplemented by the analysis of Mayeske (1969) and finally, the present version is due to Mood (1969). It is a rudimentary model of the public school system as an input-output process. All the inputs to the educational process have been classified into six major determinants: students' own abilities and attitudes, parental support, peer support, community support, quality of the school system and society's posture with respect to education. Outputs are considered as: various categories of academic achievement as well as social competence, responsibility, self confidence, creativeness, ethics and ambition. The interesting feature of this research effort is the development of index numbers or simple indicators to quantify these factors, and the approach adopted in constructing the model as a set of regression equations relating outputs to inputs. A transformation device called 'criterion scaling'[3] has been employed to introduce linearity into the model. A *striking* result demonstrated by the model is that once the rest of the factors are kept constant, the teachers' quality has a profound effect on student achievement.

The next model DYNOMOD II, known formally as the Student-Teacher Population Growth Model, is a computerized Markov-type demographic flow model which computes the number of persons in 140 distinct population groups over selected intervals of time. These population groups are cross-classified as to sex, race (2 categories), age (6 categories), educational status (3 levels of students and teachers respectively, as well as elementary and secondary school dropouts), and an 'other' category. The model employs over 832 transition probabilities to cycle the population groups. One interesting feature in the DYNOMOD II model is that all inputs and parameters can be varied. It is, thus, a particularly useful device for tracing the response and impact of policy changes on the population of students and teachers, as well as a means for discovering feasible methods of policy sequencing.

The SD Two simulation model, for the School District of Philadelphia, is a generalized planning model which can be modified to fit a district (or a county) by use of appropriate data and parameters. It permits the exploration of the effect of alternative educational programme mixes or plans on both achievement and on the consumption of resources—teachers, funds, space and equipments. The SD two model is designed so that certain areas (e.g. poverty areas) of a district may have first call on a selected programme. The level of aggregation is sufficiently detailed to provide flexibility during its use.

Although the Stone model is designed for the educational system of England and Wales, and the Thonstad model for the Norwegian educational system, both are based on the identical assumption that human flows can be viewed as a dynamic input-output process. To examine the flow through the educational system, in both models, the student population is divided into a number of age-groups and the activity 'at school' is disaggregated into a number of learning activities. While the Thonstad model uses higher order transition matrices for projection purposes, it is interesting to note that by viewing the spread of anxiety for schooling as a type of infection, the Stone model suggests the possibility of

3. It is a technique for scaling questionnaire item responses so as to maximize their correlation with an external criterion. For details, see Beaton (1969).

using the 'theory of epidemics' to modify the transition proportions for projection purposes. Thonstad has used his model to derive a series of long-term consequences of the present-day transition proportions and has discussed the problem of educational expansion to meet the growth in demand for educated manpower. The development of 'a system of demographic accounts'—a method of integrating and presenting information on the human stocks and flows of a country which can be related to its national accounts—is an important contribution stemming from the Stone model.

In brief, a marked weakness common to most of the comprehensive models is their failure to introduce capacity constraints explicitly. Although Thonstad has pointed out the importance of this complex problem, he has limited his attention to a discussion of possible ways of embodying it in his model. A further reference to this topic will be made during the subsequent discussion of partial educational planning models.

Partial Educational Planning Models

With the tremendous recent expansion of higher education and the rising levels of dissatisfaction and disaffection experienced by students, faculties, trustees and politicians, it is not surprising to find that most of the models concentrate upon the operation and planning of higher education and its institutions. In the following sections let us first consider the large-scale computerized models belonging to this category and then limit the discussion to some of the particularly interesting or attractive partial models listed in the Table.

The pioneering effort in large-scale university simulation models is the CAMPUS model, developed by Judy and Levine (1965). Originally build for the Faculty of Arts and Science at the University of Toronto, the CAMPUS model has been subsequently expanded and generalized to be applicable virtually to any college or university. The model, based on systems analysis, is capable of performing a variety of simulations for the user through its elaborate input and output routines. The model has been constructed so that most of the data needed are normally collected by institutions. By defining the institution's various activities at a desired level of aggregation, the user has the option of controlling the amount of data required by the CAMPUS model. This activity concept gives CAMPUS its great flexibility to handle practically any level of aggregation or disaggregation. "CAMPUS in its latest form is the most detailed of the educational planning models currently available."[4]

The JCL 3W model is an extension of the CAMPUS model to a very disaggregative level and is employed for planning at the University of Toronto's Faculty of Medicine. It has been reported that the inputs—staff, space, patients and supplies—of the Faculty of Medicine are currently calculated directly by the simulation of the JCL 3W model.[5]

Although not as flexible as the CAMPUS model, the MSU model developed at and for the Michigan State University is intended as an aid in achieving an optimal utilization of a University's basic resources of personnel, space and technological equipment under available financial support in the production of degree programmes, research and public or technical services. Incorporated

4. Weathersby & Weinstein (1970), p. 14.
5. Judy (1969), p. 192.

with a course credit distribution matrix and designed along the lines of linear systems, the MSU model includes dynamic propagation, such as the Markovian student flow module, and equilibrium constraints of an input-output nature. Since the parameters of the model are estimated from the past data, the effects of many important controllable variables are lost in the parameters of the model. Some of the more attractive features of the MSU are its amenability to decomposition, or suboptimization by sector, and an analysis of the sensitivity of outputs to varying parameters.

The O'Brien model is to evaluate educational policy related to location and concentration of school plants within an urban environment. Based on probability analysis, regression and linear programming and taking into account location and capacity of existing school plants, the model locates new plants, assuming that the distribution of students by their characteristics and places of residence are known.

The model for projecting enrolments and degrees awarded in universities by Gani (1963) has initiated a series of studies dealing with student flow and attendance. Oliver (1968), Marshall and Oliver (1970) and Marshall, Oliver and Suslow (1970) represent a series of increasingly sophisticated Markov models. On consideration of the units of work to be completed by undergraduates in successive semesters, the latter two works provide theoretical formulae to explain the attendance pattern of undergraduates at the Berkeley Campus of the University of California.

Although the first order Markov property is used for mathematical elegance and computational ease, its limitations in representing reality are well understood. In addition to the work of Armitage *et al* (1969) discussed above, the capacity models of Clough (1971), Harden and Tcheng (1971), McReynolds (1971) and Menges and Elstermann (1971) attempt to overcome some of these limitations. The focal point of these models is the university system. In the works of Clough and McReynolds, the focus is on the problem of selecting students under capacity constraints when their performance at college is a function not only of academic ability but also of ethics and social factors. Harden and Tcheng attempt to forecast enrolment distributions at university departments when enrolment ceiling are imposed, on the basis of a two-step Markovian model. Their experiences in utilizing the model at Illinois State University have been favourable. A more comprehensive type of capacity model has been developed by Menges and Elstermann for German University management which explicitly takes into consideration 'the time structure of the admissions optimization problem, making evident the close connection between admissions management and short-term planning. The admissions optimization problem is generalized to the problem of determining an optimal admission policy which is shaped in such a way dynamic programming techniques are applicable.' The model is thus a linear-dynamic model, complete with student flow matrix, participation matrix and capacity and faculty requirements. In brief, this is an example of an interesting and potentially useful academic planning model of the capacity constraints of a university with little or no analysis of costs, policy alternatives or benefits, and finally, an objectve function defined over inputs rather than outputs.

The analysis of Perl (1971) allows us to evaluate the viability of viewing the educational system as a production process. This model is thus particularly interesting to economists. On consideration that inputs to this process include the time and effort each student brings to the educational process, the quality of the faculty and facilties available to each student at the college attended, and the quality of the other students in the college attended, and that the primary output of this process is an increase in the student's stock of knowledge and skill, rather simple functional relationships between these inputs and outputs are postulated and estimates are obtained using multiple regression analysis. By analysing the data on about 3000 students attending 200 different colleges in the U.S., Perl has been able to demonstrate many significant results which have implications for both public and private educational decisions.

Finally the Reisman-Taft model attempts to study the production of Doctorates, Masters' and Baccalaureate degrees and their feedback into higher education. By separating out the students and the faculty subsectors within the academic sector and doing this at all levels of educational attainment, the model delineates the various possible paths for population shifts between the levels of attainment. Inflows as well as outflows of foreign nationals at various levels of attainment have also been included. The model is based on over 200 non-linear difference equations which basically take into account, at any instant of time, all of the inflows, outflows and rates of accumulation within a sector. A particularly interesting feature in the model is that it embodies the concept of a dynamic loop in which the feedback, the delays, the need for students and some of the functional relationships postulated are all combined to regulate a 'valve.' Although at present the model is restricted to the engineering profession, it is expected to be generalized to include several interrelated professions, and eventually all areas of higher learning !

The overall comparison of these partial educational planning models is postponed to the next section where we shall discuss in its entirety the major analytical weaknesses in the educational planning models brought to light by this structural analysis, with a view to pointing out their possible future development.

Possible Development of Educational Planning Models

First and foremost the Table highlights the existing operational gap between the construction of models and their usage by decision makers. Only very few models, out of those under investigation, are presently operational. Viewed from the angle that educational planning models are of very recent origin, this state of affairs should not discourage the model builders; they should proceed to rectify the existing defects in the models. If model building in educational planning is compared with similar activity in economics, it is safe to conclude that the experience in the former is still in its infancy.

Underlying many of the models is the assumption that there exist one or more recognised decision makers with the appropriate set of controls at their disposal. But looking at educational institutes, for example, one observes a complex bureaucracy with a hierarchy of decision makers, each with his own preferences and with a varying degree of control. Thus, models should in corporate the bureaucratic structure as a part of the system, and planning and allocation should be viewed as hierarchical group decisions, perhaps with the aid of the 'theory of games'[6] and the 'Delphi technique.'[7]

The goal of modelling the educational system in an explicit and quantitative way is to help in reaching at rational decisions. Optimization models, particularly, are formulated on an objective function, whether the models aid decision making by its simulation or not. A common objective function is the minimization of costs or resources needed. While cost is obviously a necessary criterion, it alone is not sufficient, specially within the educational sphere. Such factors as academic achievement concomitant with social competence, creativeness and ethics need to be considered within the objectives. These factors, although far from quantitative, can be meaningfully measured by index numbers or by sample indicators, as was done in the Coleman-Mood-Mayeske model. Objective functions which consider both costs and these factors through such index numbers will not be more valid, but the decisions reached will also be more meaningful and widely acceptable. The Arnold-McNamara model and the O'Brian model are some of the results of attempts in this direction. Even then, the objective function in the former considers only the quantitative output of graduates. The latter, on the other hand, considers in terms of broad factors the effectiveness of decisions but it is restricted to the location and size of urban schools.

Another weakness, in the postulation of objectives, is the common adoption of linear objective functions. Linearization of the objective function, of course, will result in convenient ways of solving the mathematical programming problem. But one should examine whether the decision makers' objective function is always linear. It is true that, at this early stage of educational planning model building, there is a lack of knowledge about quantification of different educational measures. Let us repeat that one such problem of measuring achievements has been successfully resolved by the Coleman-Mood-Mayeske model. The use of a linear objective function, despite its simplicity and mathematical elegance, should not always replace a quadratic function, except when it is rather unrealistic to search for a feasible solution with the quadratic objective function.

The above discussion also goes to point out the necessity of establishing coherent communication between the model builders—mainly comprising system analysts, statisticians and programmers—and the educators and sociologists. By such communications, the model builders should be able to clarify the issues facing them; in addition, model builders will be able to pose questions to the educators which will demand further experimental research. Such communications would also help to bridge the operational gap mentioned earlier.

As the construction of educational planning models progresses it has been felt that simple and straightforward mathematical techniques are not adequate in representing the complex functioning of the educational system. The

6. It seems possible to analyse institutional decision-making by using theory of games: see Luce and Raiffa (1957), Chapter 14 in particular.

7. This is a procedure which offers certain advantages in the systematic use of expert opinion and in general it has these features: anonymity controlled feedback and statistical group response. For a comprehensive review and an annotated bibliography, see Pill (1971).

computable models which facilitate speedier and better projections are of limited use. The system of planning which has probably existed and which has certainly been advocated by some writers in which projections are made in a decision-less vacuum and decisions are made in some undefined way with a few projections as relevant evidence has been condemned recently.[8] In fact, even the above structural analysis reveals that many of the models are of this type; they fail to incorporate simultaneously the feedback nature of the educational system and the necessary decision making elements. "The kind of models vitally needed for educational planning," Alper (1970) emphasizes, "are those which explicitly couple with feedback the decision making controller to that part of the educational system which is to be controlled so as to avoid the analyst providing information for policy decisions which when taken invalidate the analysis."[9]

These considerations stress the necessity of building educational planning models based upon systems analysis and control theory. While the tools and techniques of operations research are for the most part directed towards the solution of well defined problems, systems analysis is directed more towards the solution of ill-defined problems. Recently it has become fashionable to use the terminology 'operations analysis' to bring both operations research and systems analysis under a single roof. 'Operations analysis of education' seems to mean using the best of each of the fields of *operations* research and systems *analysis* to investigate those problem areas in education which are subject to quantitative analysis. Since a 'system' is a set of interacting components subject to various inputs and producing various outputs, systems analysis means an approach to problems of decision making which proceeds by ascertaining objectives, determining constraints, elaborating alternatives and estimating the costs, benefits and risks of feasible alternatives. Control theory also centres on the study of systems, but goes beyond the boundaries of systems analysis. Suppose it is given the task of controlling a system about which not everything is known initially, one can try to improve performance over time by testing and experimenting with different kinds of control actions, thus 'learning' more about the system. In control theory this is known as an 'adaptive control process'[10] since both control and learning are involved. Thus, under the theory of adaptive control processes, one is able to deal with uncertainties in the system in a tractable way. It is worth recalling that during the discussion of comprehensive externally-linked models, it was pointed out that the treatment of uncertainties within that category of models has been weak.

However, to advocate that the techniques of systems analysis and control theory need to be embodied in future educational planning models does not mean that these techniques have not been incorporated in the existing models. For that matter, the CAMPUS model and several others are based on systems analysis. The Reisman-Taft model raises the feedback issue and it is an attempt to use control theory. But what is intended here is to stress the necessity of using these techniques, particularly the theory of adaptive control

8. Armitage *et. al.*, (1969) p. 114
9. Alper (1970), p. 205.
10. For authoritative accounts, see Bellman (1961) and Fel'dbaum (1965). Murphy (1965) has demonstrated the usefulness of the theory of adaptive control processes to examine economic systems.

processes, to a greater extent in future. This is not an easy task because of the highly complex mathematics involved and the lack of detailed knowledge about the educational system itself.

We must at this point also discuss the possible development of educational planning models for developing countries. It was observed that except in few cases—the Adelman model, the Asian model, the Bowles model, the Correa-Tinbergen-Bos model, and the Kleindorfer-Roy model—most of the effort of model building has been concentrated on the educational systems or its sectors in developed countries. Davis (1966) has illustrated and discussed the models, schemata and methods for handling the general components of educational planning within human resource planning in developing countries. However, the relevance of operations research and systems analysis to educational planning in developing countries has been less well explored.

The main need in developing countries is for simple and direct solution to practical problems. Models based upon readily available data or on data that can be collected rapidly and inexpensively are clearly in the forefront. The more sophisticated the model, the more highly involved and expensive is the data base. But coming to the other end of the scale, the simpler the model, the more limited its use. Nevertheless, what are urgently needed by developing countries are those models—even if they are of limited use—which provide foundations for decision making in useful areas, like resource allocation, minimization of costs, location of schools, organization of curriculum and so forth. It is not difficult to visualize that some of the existing models, even with their admitted limitations, can be applied with suitable alterations and adjustments to areas of specific need in the educational systems of developing countries. It is sensible, at present, for individual developing countries to examine the present range of educational planning models from a practical viewpoint and choose those that can be used effectively with the available data. Meanwhile individual countries can aim at improving and expanding their own educational 'data banks' so that in the future more detailed and sophisticated computerised educational planning models can be built.

Finally it seems appropriate to point out to the model builders in developed countries that there is much scope for development of educational planning models to fit the socio-economic and ethical environments of developing countries. The difference and variability in background factors between developed and developing societies should prove an added challenge to the model builders.

Summary

With the near completion of the first decade of the educational planning model-building exercise, the extent, diversity and depth to which it has spread seems remarkable. Out of the large number of educational planning models now available, about forty were selected for discussion in this paper, and a structural comparison was carried out by using a classification framework. The major analytical weaknesses in the educational planning models brought to light by the structural analysis were then discussed in order to point out the possible future development of models. During this discussion the following features were highlighted: (a) the existing operational gap between the construction of models and their usage by the decision makers; (b) the necessity of accommodating the bureaucratic structure as a part of the system; (c) the usefulness of broadening the objective function to include not only the cost and

resources but also such factors as academic achievement, social competence, creativeness and ethics; (d) the relaxation of the linearity of the objective function; (e) the profitability of incorporating the theory of adaptive control processes; and (f) the greater emphasis required for modelling the educational systems in developing countries. In conclusion, it could be added that this kind of model-building distracts attention from issues to methodology and a challenge to model-builders is to demonstrate how their formal methods are superior to the current non-formal practice.

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