A LINEAR INTEGER PROGRAMMING APPROACH TO FIND AN OPTIMUM SCHEDULE

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Introduction

Nurse scheduling is a difficult and time-consuming task. The schedule should provide the day to day shift assignment for each nurse for a specified period of time while satisfying specified requirements in the best possible way and taking into consideration the requests made by the nurses. This paper presents an optimization technique for nurse scheduling by developing and solving a linear integer programming model. The specific objectives of the report are to reduce the workload of the head nurse in preparing a roster, reduce total cost of the hospital management regarding Over Time (OT) hours, balance the workload of the nurses, and maximize nurse satisfaction with the roster.

This model is tested by applying to a ward in Anuradhapura Teaching Hospital. The model is solved using Microsoft Excel Solver which uses Simplex Algorithm. This scientific approach in nurse scheduling takes a considerably less computational time than the current method implemented by the hospital to find the optimal nurse schedule satisfying the requirements of the hospital as well as requests of the nurses.

Methodology

Shift patterns

The hospital maintains three shifts per day: Morning shift, Evening shift and the Night shift. If a nurse is on duty, he or she must be assigned to one of the above shifts. Shift patterns have been made considering several different factors; every nurse should be assigned to one and only one shift pattern, after consecutive 16 or 17 days a nurse must work three consecutive night shifts, the minimum demand for nurses should be fulfilled for every shift of the day, every nurse should be worked at least 40 hours per week and maximum of 40 hours are considered as over-time (OT) hours.

Preference cost

For each nurse i and each shift pattern j, preference cost is calculated.

(i) Night shifts worked by the nurses in the previous month (ii) Requests made by the nurses (iii) Working history of the nurses (iv) The pregnancy of the nurses are the considerable factors when calculating the preference cost. The sum of the preference costs should be minimized.

Mathematical model

By considering the above factors, the model can be formulated as the following zero-one linear integer programming problem: (Aickelin and Dowsland, 2004)

Notations:

$i = 1,\ldots, n$ nurse index , $j = 1,\ldots, m$ shift pattern index

$k = 1,\ldots,90$ (1,...,30 morning shifts, 31,...,60 evening shifts and 61,...,90 night shifts)

p_{ij} : Preference cost of the i^{th} nurse working in the j^{th} shift pattern

$$a_{jk} = \begin{cases} 1, \text{ if the } j^{th} \text{ shift pattern covers the } k^{th} \text{ shift} \\ 0, \text{otherwise} \end{cases}$$

M_k, E_k, N_k : Number of Nurses required on the k^{th} morning, evening, night, respectively.

$$x_{ij} = \begin{cases} 1, \text{if the } i^{th} \text{ nurse works the } j^{th} \text{ shift pattern} \\ 0, \textit{otherwise} \end{cases}$$

Constraints

- Since every nurse works one and only one feasible shift pattern:

$$\sum_{j=1}^{m} x_{ij} = 1 \quad , \quad \forall i$$

- Since the total number of nurses available for each shift of the day should be greater than or equal to the required number of nurses to that shift:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} a_{jk} x_{ij} \geq M_k ,$$

$\forall k \in$ *morning shifts*

$$\sum_{i=1}^{n}\sum_{j=1}^{m} a_{jk} x_{ij} \geq E_k ,$$

$\forall k \in$ *evening shifts*

$$\sum_{i=1}^{n}\sum_{j=1}^{m} a_{jk} x_{ij} \geq N_k ,$$

$\forall k \in$ *night shifts*

Objective function

- The objective is to minimize the sum of preference costs of the nurses.

Minimize $\sum_{i=1}^{n}\sum_{j=1}^{m} p_{ij} x_{ij}$

Data collection and pre-processing

The model was aimed to the ward no: 06 in Anuradhapura Teaching Hospital. According to the case study, mathematical model is as follows:

Minimize $\sum_{i=1}^{13}\sum_{j=1}^{15} p_{ij} x_{ij}$

Subject to: $\sum_{j=1}^{15} x_{ij} = 1$,

$i = 1,2,\ldots,13$

$$\sum_{i=1}^{13}\sum_{j=1}^{15} a_{jk} x_{ij} \geq M_k ,$$

$k = 1,2,\ldots,30$

$$\sum_{i=1}^{13}\sum_{j=1}^{15} a_{jk} x_{ij} \geq E_k$$

$, k = 31,32,\ldots,60$

$$\sum_{i=1}^{13}\sum_{j=1}^{15} a_{jk} x_{ij} \geq N_k ,$$

$k = 61,62,\ldots,90$

$x_{ij} = 0$ or 1, $i = 1,\ldots,13$, $j = 1,\ldots,15$

Computational Results

Considering the time taken to prepare the roster, computer program based on Integer Linear Program is more time effective. Computer program takes only few seconds to find an optimal schedule. For manual system, head nurse spends a lot of valuable time to complete this task. In fact, the manual method is a trial and error method. The computational results are presented in Table 1.

Table 1. Computer result for nurse roster

Pattern	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Nurse1	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1
Nurse2	0	0	0	0	0	1	0	0	0	1	0	0	1	0	1
Nurse3	0	1	0	0	0	1	0	0	1	0	0	0	1	0	1
Nurse4	0	1	0	0	0	0	0	0	1	0	0	0	1	0	1
Nurse5	0	0	0	0	0	0	1	0	1	0	0	1	0	1	1
Nurse6	0	1	0	0	0	0	1	0	0	0	0	1	0	1	1
Nurse7	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
Nurse8	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1
Nurse9	0	0	1	0	0	0	0	0	0	0	1	0	1	1	1
Nurse10	0	0	1	0	0	0	0	0	0	1	0	1	0	1	1
Nurse11	0	0	1	0	0	0	0	0	0	1	0	1	0	1	1
Nurse12	0	0	1	0	0	0	0	0	0	1	0	1	0	0	1
Nurse13	0	0	1	0	0	0	0	0	0	1	0	1	0	0	1

We have developed a spreadsheet and have used Excel Solver to obtain the optimum results.

Conclusion and Recommendation

We have investigated the use of Integer Linear Programming approach using Excel Solver Add-In, and applied it to nurse scheduling problem which is highly constrained real-world problem.

At present, government hospitals in Sri Lanka use manually prepared rosters. Our attempt was to develop computer based system while taking into account the requirement of the nurses, as well as the rules and regulations in the hospital. Most importantly, this system would release the work load of the head nurse of the ward, because of the less time consuming when compared to the manual method, which is a trial and error method. Since the proposed optimal schedule reduces the Over-Time hours of the nurses. Taking these into account, the hospital management can cut down Over-Time cost. The proposed method to prepare optimal nurse schedule is more effective and efficient.

References

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